

OPTIMAL GENERALIZED LOGARITHMIC MEAN
BOUNDS FOR THE GEOMETRIC COMBINATION OF
ARITHMETIC AND HARMONIC MEANS

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Abstract. In this paper, we answer the question: for $\alpha \in (0, 1)$, what are the greatest value $p = p(\alpha)$ and least value $q = q(\alpha)$, such that the double inequality $L_p(a, b) \leq A^\alpha(a, b)H^{1-\alpha}(a, b) \leq L_q(a, b)$ holds for all $a, b > 0$? where $L_p(a, b)$, $A(a, b)$, and $H(a, b)$ are the p -th generalized logarithmic, arithmetic, and harmonic means of a and b , respectively.

Key words: Generalized logarithmic mean, arithmetic mean, harmonic mean.