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Joint-Life Insurance Premium Model Using Archimedean Copula: The Study of Mortality in Indonesia

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Abstract. Joint-life insurance pays a sum insured when the first death occurs. This insurance has a case based on the order of exit from the cohort, namely joint life and last survivor. The former means that one of the insured leaves the cohort, while the latter means the last member of the insured has left his or her cohort. For some reasons of simplicity, the insurance premium is usually calculated with the assumption that the husband and wife are mutually independent. However, this assumption is considered unrealistic. Couples are open to the same risks, hence explaining joint survival model should involve dependence structures between the distribution of spouse mortality. In line with this, to understand the dependence structure of multiple random variables, the approach used is Copula. In this context, Copula relates the marginal distribution function of these variables to the joint life distribution. One of the advantages from Copula is that the random variables do not have to come from the same distribution, hence Copula is considered good enough to explain the dependence of the mortality rate between husband and wife. This study aimed to develop a joint survival model for calculating joint life insurance premiums using the concept of Archimedean Copula to discover the minimum premium value by conducting the following steps: first, identifying the marginal distributions of mortality for genders using Indonesian Mortality Table IV (TMI/ *Tabel Mortalitas Indonesia* IV); second, Archimedean copula function-based constructing survival models that captures the relationship between these variables; third, setting dependency parameter θ; fourth, calculating the joint life premium using Archimedean copula based survival modeled for each correlation dependency level; and carrying out optimization to find the minimum premium value.

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This can be achieved by formulating the problem as an optimization problem, considering an objective function that yields the lowest premium till satisfying the financial requirements of the insurance company.

Key words and Phrases: Archimedean Copula, Joint-Life Insurance, Mortality Table, Premium Model.

1. INTRODUCTION

Life insurance is protection against the risk of a person's life, where traditionally a claim occurs if one insured person (single life) dies. However, over time, insurance that covers human lives has developed. Based on the insurance type, the claim can occur when the insured contracts a critical illness, named critical illness insurance. Based on the number of insured person, the insured in classic insurance has only one individual, but now it has been developed with more than one insured person, named joint life insurance, generally intended for married couples. The case of joint insureds is known as multiple life. There is a status in the case of multiple life based on the order of exit from the cohort, namely joint life and last survivor status. Joint life status means that one of the insured parties has left the cohort, while last survivor means that the last person has left the cohort after all the insured members have left the cohort. When calculating annuities or insurance products for multiple life, actuaries need a survival model for age pairs [1]. For reasons of simplicity, such insurance premium is usually calculated on the assumption that the husband and wife are mutually independent. However, according to Deresa, et al. [2], the assumption of independence is considered unrealistic. According to Denuit, et al. [3] and Dufresne, et al. [4], couples are open to the same risks, including common exogenous shock, and have the same lifestyle and socioeconomic characteristics. In addition, Parkes, et al. [5] and Ward, et al. [6] stated that the medical literature offers some discussion about 'Broken Heart' syndrome causing a temporary increase in the death rate of partners who have died. In addition to dependency between husband and wife on the risk of mortality, it turns out that in a husband-and-wife relationship, the risk of morbidity will also influence the level of infection of the partner. In the study conducted by Hippisley-Cox [7] to 8,386 married couples (16,772 individuals) from 29,014 participants aged 30-74 years, it was concluded that people with certain diseases increased their partner's risk of disease by at least 70%. It is clear that mortality and morbidity levels have a dependent relationship between a husband-and-wife relationship.

In line with this, Frees, et al. [8] introduced the concept of Copula as a tool to explain the dependent relationship of several random variables. Zhou and Ji [9] modelled mortality dependence using Vine Copula. Zhu, et al. [10] modelled multi-country longevity risk with mortality dependence. Even though, some previous studies on determining mortality have been conducted by researchers using some models every 3-5 years, in Indonesia, this kind of studies is still few. Thus, this study was conducted using the 2019 Indonesian Mortality Table IV (TMI/

Tabel Mortalitas Indonesia IV) applying the latest tables as a basis for calculating insurance risk for life insurance companies in Indonesia. Copula is a function that connects univariate marginals into a multivariate distribution form. Several applications were explored, including estimating joint-life mortality. Copula family known and often used in the insurance sector is the Archimedean Copula. This Copula family is considered more flexible and simpler. This study used the Archimedean Copula family to model joint life, namely Clayton Copula, Gumbel Copula, and Frank Copula.

2. RESEARCH METHODS

2.1. **Future Lifetime.** The remaining age for someone aged x is also called future lifetime, namely $x - X$, written as $K(x)$. An illustration of the remaining age distribution is shown in Figure 1.

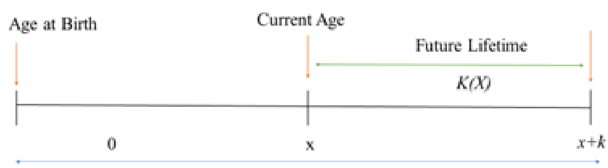


FIGURE 1. Illustration for Individual Future Lifetime

The remaining age distribution $K(x)$ can be expressed as the conditional probability that someone will leave their cohort. The formulation for future lifetime distribution function can be written as follows.

$$
\begin{aligned}
{}_kq_x &= Pr(K(x) \leq k) = Pr(X \leq x + k | X > x) && (1)\\
&= \frac{Pr(x < X \leq x + k)}{Pr(X > x)} = \frac{F(x + k) - F(x)}{S(x)} = \frac{S(x) - S(x + k)}{S(x)} && (2)\\
&= 1 - \frac{S(x + k)}{S(x)} = 1 - \frac{S(x + k)}{S(x + k - 1)} \frac{S(x + k - 1)}{S(x + k - 2)} \cdots \frac{S(x + k)}{S(x)} && (3)\\
&= 1 - \prod_{i=1}^{k} p_{x+i} = 1 - \prod_{i=1}^{k} (1 - q_{x+i}). && (4)
\end{aligned}
$$

${}_kq_x$ denotes as probability of someone with the age of x who will leave the cohort at the age of $[x; x + k)$ interval, and ${}_kp_x$ represents the complement. ${}_kp_x$ refers to probability of someone with the age of x who will survive between $[x; x + k)$ interval.

2.2. **Multiple Life.** Multiple life is a generalization of the survival distribution for more than one type of group member. It is a combined function of more than one age. If the age of men and women until death is expressed as a random vector

(X, Y), then the survival distribution and combined function can be expressed as follows:

$$F(x, y) = Pr(X \leq x, Y \leq y).$$
$$S(x, y) = Pr(X > x, Y > y).$$

A status that lasts as long as all members are alive and ends if a member dies is known as joint life. This status is expressed by (X, Y) where X represents the age of the husband's group members and Y represents the age of the wife's group ones, while K represents the remaining time until the status ends. Thus, the random variable can be expressed as follows [1], [11].

$$K(xy) = \min\{K(x), K(y)\}.$$

With the remaining life distribution function, the joint life status is expressed as follows.

$$\begin{aligned} F_{K(xy)}(k) &= Pr(\min\{K(x), K(y)\} \leq k) && (5)\\ &= Pr(K(x) \leq k \cup K(y) \leq k) && (6)\\ &= Pr(K(x) \leq k) + Pr(K(y) \leq k) - Pr(K(x) \leq k, K(y) \leq k) && (7)\\ &= {}_kq_x + {}_kq_y - F_{K(x)K(y)(k,k)}. && (8)\end{aligned}$$

Furthermore, another status in the case of multiple life is last survivor where the last member dies last after the others. The random variable for this status is expressed as follows.

$$K(\overline{xy}) = \max\{K(x), K(y)\}.$$

Distribution function for joint-life status is described as follows:

$$\begin{aligned} F_{K(xy)}(k) &= Pr(\max\{K(x), K(y)\} \leq k) && (9)\\ &= Pr(K(x) \leq k, K(y) \leq k) && (10)\\ &= F_{K(x)K(y)(k,k)}. && (11)\end{aligned}$$

Therefore, relation between last survivor and joint life relationship is as follow:

$${}_kq_{\overline{xy}} + {}_kq_{xy} = {}_kq_x + {}_kq_y. \quad (12)$$

2.3. **Archimedean Copula.** Copula is a function that can use several marginal distributions to become a joint distribution. It is a useful approach to understand the dependence structure of several random variables. The concept of Copula was first introduced by Sklar in 1959 [12] whose advantage is that the marginals do not have to be the same [13].

Definition 2.1. *A 2-dimensional Copula, C, is a multivariate distribution function, F, of random variables, $K(x), K(y)$, with distribution function of each marginal, ${}_kq_x, {}_kq_y$ is uniform distributed $(0, 1)$. This Copula function has a domain $[0, 1]^2$ and a range $[0, 1]$, which is expressed as $C : [0, 1]^2 \to [0, 1]$.*

Archimedean Copula is a class of Copula that is often used in the fields of insurance and finance [13]. Generally, the form of Archimedean Copula is as follows:

$$C({}_kq_x, {}_kq_y) = \varphi^{-1}(\varphi({}_kq_x) + \varphi({}_kq_y)) \tag{13}$$

with $0 \leq {}_kq_x, {}_kq_y \leq 1$, $C({}_kq_x, {}_kq_y)$ is Archimedean Copula, and φ is a generator function of C with properties $\varphi(0) = \infty$ and $\varphi(1) = 0$, therefore $\varphi^{[-1]} = \varphi^{-1}$ is inverse of generator function. Copula Archimedean is formed with this generator $\varphi_\phi(s) = (s^{-\phi} - 1)$, with the inverse function is $\varphi_\phi^{-1}(s) = (s+1)^{-\frac{1}{\phi}}$.

The remaining age distribution function for last survivor status can be formed using Copula, more specifically the types of Archimedean Copula with the function stated as follows:

Copula	Function
Clayton	$C({}_kq_x, {}_kq_y) = ({}_kq_x^{-\theta} + {}_kq_x^{\theta} - 1)^{-\frac{1}{\theta}}$
Gumbel	$C({}_kq_x, {}_kq_y) = exp\{(-\left[(-ln_kq_x)^\theta + (-ln_kq_y)^\theta\right])^{\frac{1}{\theta}}\}$
Frank	$C({}_kq_x, {}_kq_y) = -\frac{1}{\theta} ln\left[1 + \frac{(e^{-\theta_kq_x} - 1)(e^{-\theta_kq_y} - 1)}{e^{-\theta} - 1}\right]$

Table 1. Types of Archimedean Copula family

The Table 1 refers to distribution function of last survivor status, while the relation between joint life and last survivor has been derived in Section 2.2. These three Copulas are applied to mortality data to determine joint-life insurance premiums for married couples. The application of ${}_kq_{\overline{xy}}$ in Frank Copula [1] has been derived in the rest member of Archimedean Copula models with their own model constructed the same way, and the whole result is served in Table 1.

2.4. **Premium Calculation.** Here is one of ways to calculate premium using equivalencies principles that satisfies [1], [11].

$$E[Loss] = 0.$$

In this case, the Loss random variables can be expressed as $Loss = Outflow - Inflow$.

Therefore, joint-life term insurance premium can be derived as follows:

$$E[Loss] = 0 \iff E[Outflow - Inflow] = 0 \tag{14}$$

$$\iff E[Outflow] - E[Inflow] = 0 \tag{15}$$

$$\iff A_{xy^1:\overline{n}|} - \pi_{xy^1:\overline{n}|}\ddot{a}_{xy:\overline{n}|} = 0 \tag{16}$$

and can be formulated as follows:

$$\pi_{xy^1:\overline{n}|} = \frac{A_{xy^1:\overline{n}|}}{\ddot{a}_{xy:\overline{n}|}} = \frac{\sum_{k=0}^{n-1} b_k \cdot v_{k+1} \cdot {}_kp_{xy} \cdot q_{x+k:y+k}}{\sum_{k=0}^{n-1} v_k \cdot {}_kp_{xy}}. \tag{17}$$

$A_{xy^1:\overline{n}|}$ is present value future benefit where b_k is sum insured, $\ddot{a}_{xy:\overline{n}|}$ is joint life annuity, and $v_k = \prod_{j=1}^{k}(1+i)^{-j}$ and i are interest rate.

3. DATA

The data used in this study was the Indonesian mortality table, especially the Indonesian Mortality Table IV (TMI IV). This is a mortality table compiled by the Indonesian Life Insurance Association (AAJI/ *Asosiasi Asuransi Jiwa Indonesia*) using data from 52 life insurance companies in Indonesia which can be used for the life insurance industry's needs for a reference that can describe Indonesia's mortality conditions [14]. The compiled mortality table is the result of probability estimate that someone aged x will die 1 year later, symbolized by q_x, with x for men and y for women. The estimated results of the q_x calculation on TMI IV is shown in Figure 2.

The Figure 2 shows the mortality rate at ages 41 to 85 for men presenting a faster death rate with increasing age compared to women. It can be concluded that the higher the male age level, the greater the difference between the female mortality rates. In contrast to the mortality phenomenon for ages 86 to 111, the mortality rate for men is slowing down, while for women it is getting faster, so the difference in mortality rates is getting smaller.

Apart from death can cause financial loss for the relatives being left, critical illness can also be considered a risk since if it occurs, it will cause loss. The risk of critical illness is quantified in the Indonesian Morbidity Table [15] as in Figure 3.

The Morbidity Table in the Figure 3 shows the estimated morbidity probability that a person aged x will experience serious illness within a period of up to 1 year, q_x and q_y, with x is for men and y is for women. Female morbidity rate fluctuates up and down like a wave model every 10 decades, whereas for the same period, male morbidity rate is more consistent and gradually becomes greater as age increases.

4. RESULTS AND DISCUSSION

The data in the Mortality Table previously explained was applied to the Archimedean Copulas, namely Clayton, Frank, and Gumbel Copulas. According to Central Statistics Agency (BPS/ *Badan Pusat Statistik*) [16], the majority of female started to marry at the age of 19-21 years old (37.2%) and male at the age of 22-24 years old (35.21%). With the assumption that the couples will start the peak of career in 10-15 years of working, the determination of premium calculation starts from age of 35-45 years, 45 years due to its late career phase.

From Table 2, it can be seen that when husband (x) and wife (y) are in their productive age, 35 years, the amount of Joint-Life Insurance premiums for 5 years, 10 years, and 20 years with $\theta = 1.5$ is very little respectively, namely IDR 144,189; IDR 183,247; and IDR 296,762. Meanwhile, when husband (x) and wife (y) are both approaching retirement age, 45 years, with $\theta = 0.5$, a very high premium is obtained, namely IDR 496,714 for 5 years, IDR 624,019 for 10 years, and IDR 849,580 for 20 years.

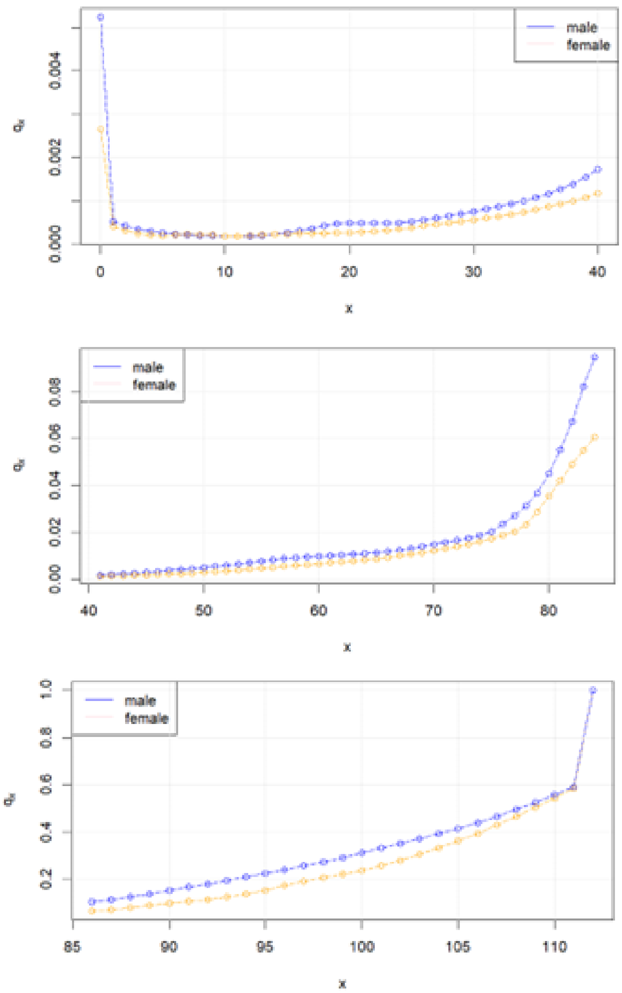


FIGURE 2. Characteristic of Indonesian Mortality Table IV (TMI IV)

In Table 3, the results using Frank Copula are slightly higher than Clayton Copula. When husband (x) and wife (y) are in their productive age, 35 years, the little premium amount is obtained with $\theta = 1.5$ for 5 years, 10 years, and 20 years, with IDR 207,262; IDR 261,622; and IDR 418,297 respectively. Likewise, when husband (x) and wife (y) are both approaching retirement age, 45 years, with $\theta = 0.5$, a very high premium is obtained, namely IDR 567,312 for 5 years, IDR 712,577 for 10 years, and IDR 969,584 for 20 years.

From Table 4, it can be seen that when husband (x) and wife (y) are both in their productive age, 35 years, the Joint-Life Insurance premium for 5 years, 10 years, and 20 years with $\theta = 0.5$, is very little, namely IDR 200,868; IDR

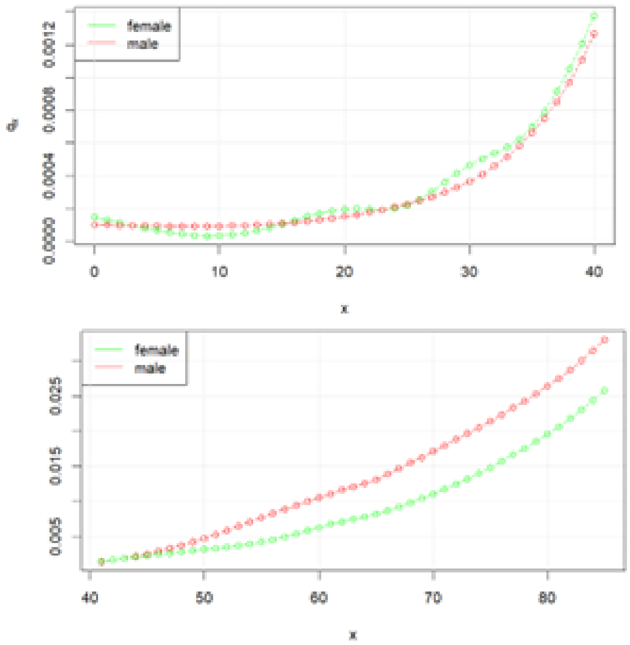


FIGURE 3. Characteristic of Indonesian Morbidity Table

Clayton Copula										
		(x)								
(y)	θ	35			40			45		
		Coverage Period (n years)								
		5	10	20	5	10	20	5	10	20
35	0.5	181184	228948	366952	364088	466429	687264	399794	501363	686552
	1.0	156903	198895	320787	314251	403059	596139	370364	465867	634386
	1.5	144189	183247	296762	287201	368415	545695	359610	453041	614012
40	0.5	219946	281057	443563	293612	379516	578215	432095	544595	751522
	1.0	189618	242519	384169	255662	331410	507077	389126	490918	674586
	1.5	173332	221697	351609	236006	306522	469821	370163	467054	638377
45	0.5	294619	373132	555962	364007	465888	683406	496714	624019	849580
	1.0	258488	327232	486535	314195	402683	593429	434518	546838	745631
	1.5	240431	303984	449856	287157	368131	543667	402386	506738	690431

TABLE 2. Result of Joint-Life Insurance Premium Calculation with Clayton Copula (Insured Sum IDR 100 million)

253,471; and IDR 404,732 respectively. Meanwhile, when husband (x) and wife (y) are both approaching retirement age, 45 years, with $\theta = 1.5$, a very high premium is obtained, namely IDR 568,351 for 5 years, IDR 714,236 for 10 years, and IDR 972,121 for 20 years. The results obtained using Gumbel Copula are not too different from Frank Copula.

From these three results, it can be seen that the husband's age (x) influences the high increase in Joint-Life Insurance premiums. This shows that the chance of death for men is higher than for women.

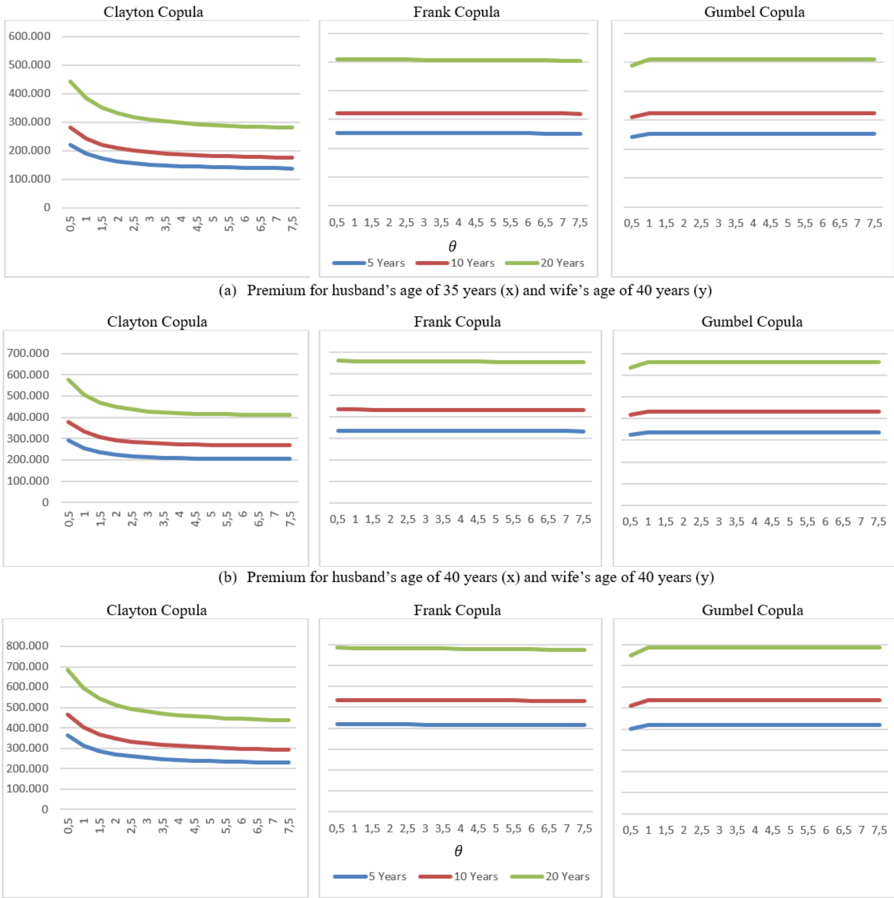
Frank Copula										
		(x)								
(y)	θ	35			40			45		
		Coverage Period (n years)								
		5	10	20	5	10	20	5	10	20
35	0.5	207336	261743	418613	290507	372697	570099	439904	550708	757474
	1.0	207301	261686	418464	290448	372599	569885	439802	550548	757189
	1.5	207262	261622	418297	290382	372490	569644	439688	550370	756866
40	0.5	252432	322685	508857	335531	433401	659305	484806	611073	845828
	1.0	252378	322594	508638	335441	433246	658989	484650	610821	845406
	1.5	252319	322492	508391	335340	433073	658632	484477	610539	844927
45	0.5	335290	425076	634640	418259	535425	783941	567312	712577	969584
	1.0	335204	424931	634339	418113	535178	783506	567060	712175	968995
	1.5	335107	424769	633998	417950	534902	783012	566777	711725	968322

Table 3. Result of Joint-Life Insurance Premium Calculation with Frank Copula (Insured Sum IDR 100 million)

Gumbel Copula										
		(x)								
(y)	θ	35			40			45		
		Coverage Period (n years)								
		5	10	20	5	10	20	5	10	20
35	0.5	200868	253471	404732	283416	363632	555233	432111	540851	741793
	1.0	207366	261793	418742	290558	372782	570284	439993	550847	757719
	1.5	207479	261979	419224	290749	373098	570996	440324	551364	758701
40	0.5	242613	309647	487314	324766	419114	636206	472975	595545	821432
	1.0	252478	322764	509047	335609	433536	659576	484941	611293	846189
	1.5	252649	323058	509755	335900	434036	660624	485444	612111	847648
45	0.5	319321	404078	602700	400750	512414	749610	548069	687563	933186
	1.0	335365	425202	634899	418387	535640	784313	567532	712928	970086
	1.5	335644	425671	635859	418860	536439	785743	568351	714236	972121

Table 4. Result of Joint-Life Insurance Premium Calculation with Gumbel Copula (Insured Sum IDR 100 million)

From Figure 4, it can be seen that the choice of husband's age (x) and wife's age (y) does not significantly influence the characteristics of the Joint-Life term life insurance premium, however the choice of Copula type in the Archimedean Copula family provided a quite different reference. Gumbel Copula could capture differences at small values of θ and then experienced convergence. Furthermore, the premium calculated using Frank Copula provided poor results in capturing changes in the premium to the Copula parameters. In contrast to the Joint-Life premium calculated using Clayton Copula, it provided a smaller premium estimate, but it was better at capturing differences in the level of dependency between married couples. When examined more deeply, the premium amount would decrease exponentially and then reach convergence to the husband's individual premium price. Additionally, when the value of θ was reduced, it would go to the Joint-Life premium price calculated under the assumption of mutual independence. From the calculation results, it is known that the assumption of dependence between husband and wife is very realistic, explained by Clayton Copula, this is in accordance with the assumptions put forward by Deresa, et al. [2].



(a) Premium for husband's age of 35 years (x) and wife's age of 40 years (y)

(b) Premium for husband's age of 40 years (x) and wife's age of 40 years (y)

(c) Premium for husband's age of 40 years (x) and wife's age of 35 years (y)

FIGURE 4. Comparison of Joint-Life Premium Calculation Results using Archimedean Copula

5. CONCLUSION

From the results obtained, it can be concluded that the calculation of Joint-Life Insurance premiums for husband-and-wife couples in accordance with the assumption of dependence is Clayton Copula. From the application of the three Archimedean Copulas, it can be seen that the husband's age (x) influences the high increase in Joint-Life Insurance premiums in accordance with the assumption used in the mortality table that the death rate for men is higher than for women. This study can be a benchmark industry for premium pricing and credit insurance. It can also be implemented in morbidity table. For further research, it is recommended to explore methods that can group criteria for husband-and-wife couples in accordance with certain parameter values θ.

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