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The Locating-Chromatic Number of Some Jellyfish Graphs

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Abstract. Let c be a proper coloring of a graph $G = (V, E)$ with k colors which induces a partition Π of $V(G)$ into color classes $L_1, L_2, \ldots, L_k$. For each vertex v in G, the color code $c_\Pi(v)$ is defined as the ordered k-tuple $(d(v, L_1), d(v, L_2), \ldots, d(v, L_k))$, where $d(v, L_i)$ represents the minimum distance from v to all other vertices u in $L_i (1 \leq i \leq k)$. If every vertex possesses unique color codes, then c is called a locating-k-coloring in G. If k is the minimum number such that c is a locating-k-coloring in G, then the locating-chromatic number of G is $\chi_L(G) = k$. In this paper, the author determine the locating-chromatic number of some Jellyfish Graphs.

Key words and Phrases: Locating-Coloring, Locating-Chromatic Number, Jellyfish Graphs.

1. INTRODUCTION

Consider a finite, simple, and connected graph $G = (V, E)$. Let c be a proper coloring of a graph $G = (V, E)$ with k colors which induces a partition Π of $V(G)$ into color classes $L_1, L_2, \ldots, L_k$. For each vertex v in G, the color code $c_\Pi(v)$ is defined as the ordered k-tuple $(d(v, L_1), d(v, L_2), \ldots, d(v, L_k))$, where $d(v, L_i)$ represents the minimum distance from v to all other vertices u in $L_i (1 \leq i \leq k)$. If every vertex possesses unique color codes, then c is called a locating-k-coloring in G. If k is the minimum number such that c is a locating-k-coloring in G, then the locating-chromatic number of G is $\chi_L(G) = k$.

Chartrand et al. [1] initially proposed the concept of locating-chromatic number of a graph in 2002. They determined the locating-chromatic numbers of familiar graphs including paths P_n, cycles C_n, double stars $S_{a,b}$, and complete multipartite graphs. They also established constraints of the locating-chromatic number based

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on the order and diameter of a graph. Additionally, Chartrand et al. [2] extended their investigation to trees where they proved that for any $k \in \{3, 4, ..., n-2, n\}$, there exists a tree T of order n with $\chi_L(T) = k$ and showed that it is impossible to have a tree T of order n with $\chi_L(T) = n - 1$.

It is noteworthy that there is no universally applicable algorithm to compute the locating-chromatic number for graphs [3]. Consequently, research on the locating-chromatic number is focused on specific classes of graphs or involves in characterizing graphs with certain locating-chromatic numbers. Various studies have explored the locating-chromatic numbers for graph obtained through the operations involving two graphs, such as the join of graphs [4], Cartesian product [5], and corona product [6]. Furthermore, researchers have identified the locating-chromatic number for several unique graph structures, such as Firecracker graphs [7], Pizza graphs [8], Origami graphs [9], Lobster graphs [10], Caterpillar graphs [11], Book graphs [12], Barbell graphs [11], Halin graphs [3], and Kneser graphs [13].

In the case of characterization, Chartrand et al. [2] provided a characterization for all connected graphs G of order $n \geq 4$ with $\chi_L(G) = 3$. Baskoro and Asmiati [14] characterized all trees having locating-chromatic number 3. Asmiati and Baskoro [15] also characterized all graphs containing cycles with locating-chromatic number 3. Arfin and Baskoro also characterized all unicyclic graphs having locating-chromatic number $n - 2$ [16] and $n - 3$ [17].

In this paper, the author is interested in exploring the locating-chromatic number of some Jellyfish graphs. The term "jellyfish" denotes a graph structure resembling the shape of a jellyfish. However, there are several articles that provide different definitions of the Jellyfish graph. The author found that there are at least three sources that use the term "Jellyfish graph" in their research, each with their own respective definitions.

(1) A Jellyfish graph defined by Lee and Lee [18], denoted as $J_{m,n}$, with the following definition.
The Jellyfish graph $J_{m,n}$ for $m, n \geq 1$ is a graph with a set of vertex $V = \{u, v, x, y\} \cup \{x_i | 1 \leq i \leq m\} \cup \{y_j | 1 \leq j \leq n\}$ and a set of edge $E = \{ux, uv, uy, vx, vy\} \cup \{xx_i | 1 \leq i \leq m\} \cup \{yy_j | 1 \leq j \leq n\}$. Figure 1 displays the structure of graph $J_{m,n}$.

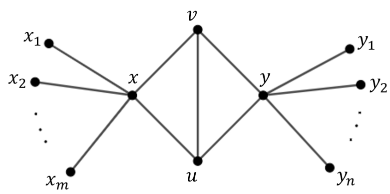

FIGURE 1. Graph $J_{m,n}$

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(2) A Jellyfish graph defined by Azka et al. [19], denoted as J_n, with the following definition.
Let C_n be a cycle graph of order $n \geq 3$ with $V(C_n) = \{v_i | 1 \leq i \leq n\}$. Let P and Q be two path graphs of order $\lfloor \frac{n}{2} \rfloor$, with $V(P) = \{p_j | 1 \leq j \leq \lfloor \frac{n}{2} \rfloor\}$ and $V(Q) = \{q_k | 1 \leq k \leq \lfloor \frac{n}{2} \rfloor\}$. The Jellyfish graph J_n is a graph obtained by connecting P and Q onto C_n by adding edges v_1p_1 and v_nq_1. Figure 2 displays the structure of graph J_n.

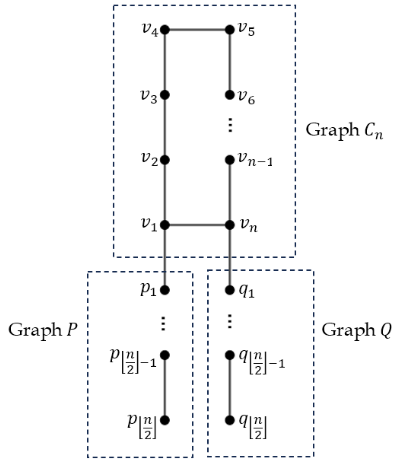

FIGURE 2. Graph J_n

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(3) A Jellyfish graph defined by Akbar and Sugeng [20], denoted as $J(m, n)$, with the following definition.
The Jellyfish graph $J(m, n)$ is a graph with a set of vertex $V = \{v_i | 1 \leq i \leq n\} \cup \{x, y\} \cup \{x_j, y_j | 1 \leq j \leq m\}$ and a set of edge $E = \{xy, xv_i, yv_i | 1 \leq i \leq n\} \cup \{xx_j, yy_j | 1 \leq j \leq m\}$. Figure 3 displays the structure of graph $J(m, n)$.

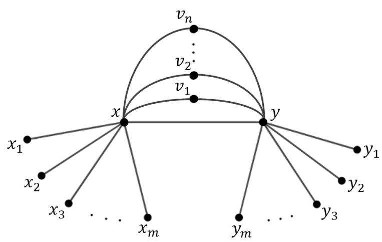

FIGURE 3. Graph $J(m, n)$

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2. BASIC PROPERTIES

This section encompasses essential attributes concerning the locating-chromatic number of graphs. Consider a connected graph $G(V, E)$ with an order of n, and v is a vertex in G. The *neighborhood* of v is defined as $N(v) = \{x \in V(G) | xv \in E(G)\}$, and the *degree* of v is defined as $deg(v) = |N(v)|$ [17]. A *leaf* is a vertex in G with degree one [17, 1]. The number of leaves adjacent to v indicates the *external degree* of v and is denoted as $d^+(v)$. The maximum value of $d^+(v)$ for each vertex v in G indicates the *maximum external degree* of G and is denoted as $\Delta^+(G)$ [17]. The following theorems and corollary are inherent to this context.

Theorem 2.1. [1] *If G is a connected graph of order $n \geq 3$, then $3 \leq \chi_L(G) \leq n$.*

Theorem 2.2. [1] *Let c be a locating-coloring in a connected graph G. If u and v are distinct vertices of G such that $d(u, w) = d(v, w)$ for all $w \in V(G) \backslash \{u, v\}$, then $c(u) \neq c(v)$. In particular, if u and v are nonadjacent vertices of G such that $N(u) = N(v)$, then $c(u) \neq c(v)$.*

Corollary 2.3. [1] *If G is a connected graph containing a vertex v with $d^+(v) = p$, then $\chi_L(G) \geq p$. Furthermore, if $\Delta^+(G) = P$, then $\chi_L(G) \geq P + 1$.*

3. MAIN RESULTS

Theorem 3.1. *The locating-chromatic number of $J_{m,n}$ for $m \geq n \geq 1$ is*

$$\chi_L(J_{m,n}) = \begin{cases} 4, & m < 3, \\ m+1, & m \geq 3. \end{cases}$$

Proof. The proof will be divided into two cases, i.e. for $m < 3$ and for $m \geq 3$.

Case 1. $m < 3$. Notably, u, v, and x form pairwise adjacent vertices, so they must be assigned with distinct colors. Therefore, $\chi_L(J_{m,n}) \geq 3$. Now, assume $\chi_L(J_{m,n}) = 3$ and let c be a locating-3-coloring in $J_{m,n}$ such that $c(u) = 1$, $c(v) = 2$, and $c(x) = 3$. However, as u, v, and y also form pairwise adjacent vertices, y must be assigned with color 3, which leads to contradiction since $c_\Pi(x) = c_\Pi(y) = (1, 1, 0)$. Therefore, it is deduced that $\chi_L(J_{m,n}) \geq 4$.

To establish an upper bound, given that $m < 3$, three possible graph exist, i.e. $J_{1,1}$, $J_{2,1}$, and $J_{2,2}$. Figure 4 illustrates these three graphs along with their minimum locating-4-coloring. Consequently, it is concluded that $\chi_L(J_{m,n}) \leq 4$.

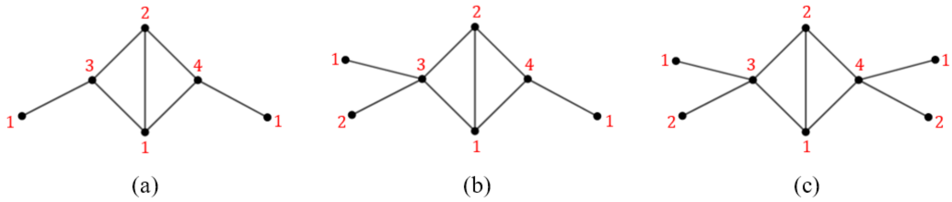

FIGURE 4. Graph (a) $J_{1,1}$, (b) $J_{2,1}$, and (c) $J_{2,2}$ with their minimum locating-coloring

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Case 2. $m \geq 3$. Note that $\Delta^+(J_{m,n}) = d^+(x) = m \geq 3$. By Corollary 2.3, it implies $\chi_L(J_{m,n}) \geq m+1$. To establish an upper bound, define a coloring c' : $V(J_{m,n}) \rightarrow \{1, 2, ..., m+1\}$ such that $c'(u) = 1$, $c'(v) = 2$, $c'(x) = 3$, $c'(y) = m+1$, $c'(y_j) = j$ for $1 \leq j \leq n$, and

$$c'(x_i) = \begin{cases} i, & i = 1, 2, \\ i+1, & 3 \leq i \leq m. \end{cases}$$

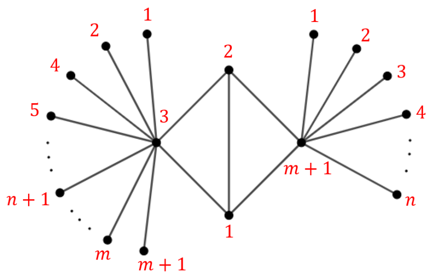

FIGURE 5. The coloring c' in $J_{m,n}$

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Let $\Pi = \{L_k | 1 \leq k \leq m+1\}$ be a partition of $V(J_{m,n})$ induced by c', where L_k containing vertices of color k. Let a and b be two distinct vertices such that $c'(a) = c'(b)$. It follows that a and b must belong to $L_1, L_2, ..., L_n$, or L_{m+1}, since L_{n+1}, L_{n+2}, ..., L_m are either singleton sets (for $m > n$) or empty sets (for $m = n$).

- If $a, b \in L_1$, then $a, b \in \{u, x_1, y_1\}$. Note that $d(u, L_2) = 1$ and $d(x_1, L_2) = d(y_1, L_2) = 2$, so $c'_\Pi(u)$ must be distinct with $c'_\Pi(x_1)$ and $c'_\Pi(y_1)$. Moreover, since $d(x_1, L_{m+1}) = 2$ and $d(y_1, L_{m+1}) = 1$, it follows that $c'_\Pi(x_1) \neq c'_\Pi(y_1)$, therefore $c'_\Pi(a) \neq c'_\Pi(b)$;

- If $a, b \in L_2$, then $a, b \in \{v, x_2, y_2\}$. Note that $d(v, L_1) = 1$ and $d(x_2, L_1) = d(y_2, L_1) = 2$, so $c'_\Pi(v)$ must be distinct with $c'_\Pi(x_2)$ and $c'_\Pi(y_2)$. Moreover, since $d(x_2, L_{m+1}) = 2$ and $d(y_2, L_{m+1}) = 1$, it follows that $c'_\Pi(x_2) \neq c'_\Pi(y_2)$, therefore $c'_\Pi(a) \neq c'_\Pi(b)$;
- If $a, b \in L_3$, then $a, b \in \{x, y_3\}$. Since $d(x, L_1) = 1$ and $d(y_3, L_1) = 2$, it follows that $c'_\Pi(a) \neq c'_\Pi(b)$;
- If $a, b \in L_{m+1}$, then $a, b \in \{y, x_m\}$. Since $d(y, L_1) = 1$ and $d(x_m, L_1) = 2$, it follows that $c'_\Pi(a) \neq c'_\Pi(b)$;
- If $a, b \in L_k (4 \leq n \leq n)$, then $a, b \in \{x_{k-1}, y_k\}$. Since $d(x_{k-1}, L_{m+1}) = 2$ and $d(y_k, L_{m+1}) = 1$, it follows that $c'_\Pi(a) \neq c'_\Pi(b)$.

Since all vertices of $J_{m,n}$ have distinct color codes, c' is a locating-coloring in $J_{m,n}$. Therefore, $\chi_L(J_{m,n}) \leq m + 1$. □

Theorem 3.2. *The locating-chromatic number of J_n for $n \geq 3$ is $\chi_L(J_n) = 3$.*

Proof. For all connected graphs, it is evident that the lower bound of its locating-chromatic number is 3, therefore $\chi_L(J_n) \geq 3$. It will be demonstrated that $\chi_L(J_n) \leq 3$ by considering two cases, i.e. for n odd and n even.

Case 1. n odd. Define a coloring $c_1 : V(J_n) \to \{1, 2, 3\}$ such that

$$c_1(v_i) = \begin{cases} 1, & i = 1, \\ 2, & i \geq 2, i \text{ is even}, \\ 3, & i \geq 3, i \text{ is odd}. \end{cases}$$

and

$$c_1(p_j) = \begin{cases} 1, & j \text{ is even}, \\ 2, & j \text{ is odd}. \end{cases} ; c_1(q_k) = \begin{cases} 1, & k \text{ is odd}, \\ 3, & k \text{ is even}. \end{cases}$$

Let $\Pi = \{L_1, L_2, L_3\}$ be a partition of $V(J_n)$ induced by c_1, where L_k containing vertices of color k. Let a and b be two distinct vertices such that $c_1(a) = c_1(b)$.

- If $a, b \in L_1$, then $a, b \in \{v_1\} \cup \{p_j | 1 \leq j \leq \lfloor \frac{n}{2} \rfloor, j \text{ is even}\} \cup \{q_k | 1 \leq k \leq \lfloor \frac{n}{2} \rfloor, k \text{ is odd}\}$. It follows that $c_{1\Pi}(v_1) = (0, 1, 1)$, $c_{1\Pi}(p_j) = (0, 1, j + 1)$, and $c_{1\Pi}(q_k) = (0, k + 1, 1)$, therefore $c_{1\Pi}(a) \neq c_{1\Pi}(b)$;
- If $a, b \in L_2$, then $a, b \in \{v_i | 2 \leq i \leq n, i \text{ is even}\} \cup \{p_j | 1 \leq j \leq \lfloor \frac{n}{2} \rfloor, j \text{ is odd}\}$. Since $d(v_i, L_3) = 1$ and $d(p_j, L_3) = j + 1$, it follows that $c_{1\Pi}(a) \neq c_{1\Pi}(b)$;
- If $a, b \in L_3$, then $a, b \in \{v_i | 2 \leq i \leq n, i \text{ is odd}\} \cup \{q_k | 1 \leq k \leq \lfloor \frac{n}{2} \rfloor, k \text{ is even}\}$. Since $d(v_i, L_2) = 1$ and $d(q_k, L_2) = k + 1$, it follows that $c_{1\Pi}(a) \neq c_{1\Pi}(b)$.

Since all vertices in J_n have distinct color codes, c_1 is a locating-coloring in J_n. Therefore $\chi_L(J_n) \leq 3$.

Case 2. n even. Define a coloring $c_2 : V(J_n) \to \{1, 2, 3\}$ such that

$$c_2(v_i) = \begin{cases} 1, & i \text{ is odd}, \\ 2, & i \text{ is even}, \end{cases} ; c_1(p_j) = \begin{cases} 1, & j \text{ is even}, \\ 3, & j \text{ is odd}. \end{cases} ; c_1(q_k) = \begin{cases} 1, & k \text{ is even}, \\ 3, & k \text{ is odd}. \end{cases}$$

Let $\Pi = \{L_1, L_2, L_3\}$ be a partition of $V(J_n)$ induced by c_2, where L_k containing vertices of color k. Let a and b be two distinct vertices such that $c_2(a) = c_2(b)$.

- If $a, b \in L_1$, then $a, b \in \{v_i | 1 \leq i \leq n, i \text{ is odd}\} \cup \{p_j | 1 \leq j \leq \lfloor \frac{n}{2} \rfloor, j \text{ is even}\} \cup \{q_k | 1 \leq k \leq \lfloor \frac{n}{2} \rfloor, k \text{ is even}\}$. Since $d(v_i, L_2) = 1$, $d(p_j, L_2) = j + 1(\geq 3,\ \text{odd})$ and $d(q_k, L_2) = k(\geq 2,\ \text{even})$, it follows that $c_{2\Pi}(a) \neq c_{2\Pi}(b)$;
- If $a, b \in L_2$, then $a, b \in \{v_i | 1 \leq i \leq n, i \text{ is even}\}$. Since $d(v_i, L_3) = i(\text{even})$ for $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$ and $d(v_i, L_3) = n - i + 1(\text{odd})$ for $\lfloor \frac{n}{2} \rfloor < i \leq n$, it follows that $c_{2\Pi}(a) \neq c_{2\Pi}(b)$;
- If $a, b \in L_3$, then $a, b \in \{p_j | 1 \leq j \leq \lfloor \frac{n}{2} \rfloor, j \text{ is odd}\} \cup \{q_k | 1 \leq k \leq \lfloor \frac{n}{2} \rfloor, k \text{ is odd}\}$. Since $d(p_j, L_2) = j + 1(\geq 2,\ \text{even})$ and $d(q_k, L_2) = k(\geq 1,\ \text{odd})$, it follows that $c_{2\Pi}(a) \neq c_{2\Pi}(b)$.

Since all vertices in J_n have distinct color codes, c_2 is a locating-coloring in J_n. Therefore $\chi_L(J_n) \leq 3$. □

Theorem 3.3. *The locating-chromatic number of $J(m, n)$ for $m, n \geq 1$ is $\chi_L(J(m, n)) = \max\{m + 1, n + 2\}$. In particular,*

$$\chi_L(J(m, n)) = \begin{cases} m + 1, & m > n, \\ n + 2, & m \leq n. \end{cases}$$

Proof. Let c be a proper coloring in $J(m, n)$. Observe that $\Delta^+(J(m, n)) = d^+(x) = d^+(y) = m$. By Corollary 2.3, we have $\chi_L(J(m, n)) \geq m + 1$. Furthermore, it is observed that $N(v_i) = N(v_j) = \{x, y\}$ for all $1 \leq i, j \leq n$ where $i \neq j$. Consequently, it follows that $c(v_i) \neq c(v_j)$. Wlog, assume $c(v_i) = i$ for $1 \leq i \leq n$. Since x and y are adjacent vertices and share adjacency with $v_i (1 \leq i \leq n)$, it is essential that x and y receive distinct colors not belonging to the set $\{1, 2, ..., n\}$. Thus, assume $c(x) = n + 1$ and $c(y) = n + 2$. This assignment leads to the observation that $\chi_L(J(m, n)) \geq n + 2$. From the observations, it can be concluded that $\chi_L(J(m, n)) \geq \max\{m + 1, n + 2\}$.

Case 1. $m > n$. It follows that $\max\{m + 1, n + 2\} = m + 1$, implying that $\chi_L(J(m, n)) \geq m + 1$. To establish an upper bound, define a coloring c_1 such that $c_1(v_i) = c(v_i)$ for all $1 \leq i \leq n$, $c_1(x) = c(x)$, and $c_1(y) = c(y)$. Then, assign colors $1, 2, ..., n, n + 2, ..., m + 1$ to all leaves of x, and colors $1, 2, ..., n + 1, n + 3, ..., m + 1$ to all leaves of y (see Figure 6).

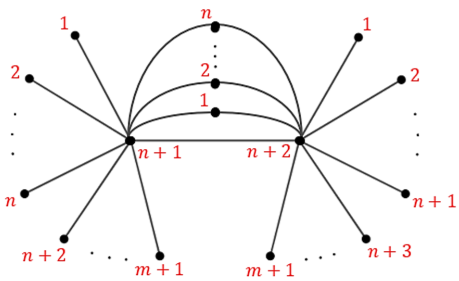

Figure 6. The locating-coloring c_1 in $J(m, n)$ for $m > n$

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Let $\Pi = \{L_k | 1 \leq k \leq m + 1\}$ be a partition of $V(J(m, n))$ induced by c_1, where L_k containing vertices of color k. Let a and b be two distinct vertices such that $c_1(a) = c_1(b)$.

- If $a, b \in L_k (1 \leq k \leq n)$, then $a, b \in \{v_k, x_k, y_k\}$. The color codes of v_k, x_k, and y_k are distinguished from the $(n+1)$-th and $(n+2)$-th entries, where $c_{1\Pi}(v_k) = (..., 1, 1, ...)$, $c_{1\Pi}(x_k) = (..., 1, 2, ...)$, and $c_{1\Pi}(y_k) = (..., 2, 1, ...)$. Hence, $c_{1\Pi}(a) \neq c_{1\Pi}(b)$;
- If $a, b \in L_{n+1}$, then $a, b \in \{x, y_{n+1}\}$. Since $d(x, L_1) = 1$ and $d(y_{n+1}, L_1) = 2$, it follows that $c_{1\Pi}(a) \neq c_{1\Pi}(b)$;
- If $a, b \in L_{n+2}$, then $a, b \in \{y, x_{n+1}\}$. Since $d(y, L_1) = 1$ and $d(x_{n+1}, L_1) = 2$, it follows that $c_{1\Pi}(a) \neq c_{1\Pi}(b)$;
- If $a, b \in L_k (n + 3 \leq k \leq m + 1)$, then $a, b \in \{x_{k-1}, y_{k-1}\}$. The color codes of x_{k-1} and y_{k-1} are distinguished from the $(n + 1)$-th and $(n + 2)$-th entries, where $c_{1\Pi}(x_{k-1}) = (..., 1, 2, ...)$ and $c_{1\Pi}(y_{k-1}) = (..., 2, 1, ...)$. Hence, $c_{1\Pi}(a) \neq c_{1\Pi}(b)$.

Since all vertices in $J(m, n)$ have distinct color codes, c_1 is a locating-coloring in $J(m, n)$. Therefore $\chi_L(J(m, n)) \leq m + 1$.

Case 2. $m \leq n$. It follows that $\max\{m + 1, n + 2\} = n + 2$, implying that $\chi_L(J(m, n)) \geq n + 2$. To establish an upper bound, define a coloring c_2 such that $c_2(v_i) = c(v_i)$ for all $1 \leq i \leq n$, $c_2(x) = c(x)$, and $c_2(y) = c(y)$. Then, given that $m \leq n$, all leaves of x and y can be assigned to colors $1, 2, ..., m$ respectively (see Figure 7).

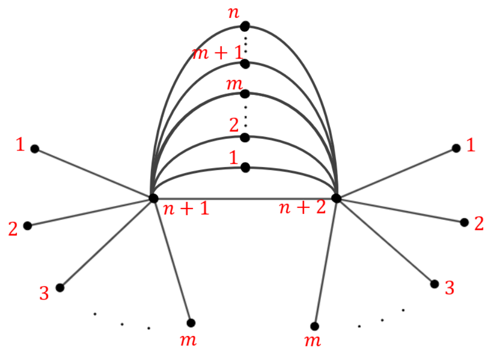

FIGURE 7. The locating-coloring c_2 in $J(m,n)$ for $m \leq n$

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Let $\Pi = \{L_k | 1 \leq k \leq n+2\}$ be a partition of $V(J(m,n))$ induced by c_2, where L_k containing vertices of color k. Let a and b be two distinct vertices such that $c_2(a) = c_2(b)$. It follows that a and b must be contained in $L_1, L_2, ..., L_m$ since $L_{m+1}, L_{m+2}, ..., L_{n+2}$ are singleton sets. Observe that $L_k = \{v_k, x_k, y_k\}$ for $1 \leq k \leq m$. For each $k (1 \leq k \leq m)$, the color codes of v_k, x_k, and y_k have 0 for its k-th entry, and are distinguished from their $(n+1)$-th and $(n+2)$-th entries, where $c_{2\Pi}(v_k) = (..., 1, 1)$, $c_{2\Pi}(x_k) = (..., 1, 2)$, and $c_{2\Pi}(y_k) = (..., 2, 1)$. Thus, $c_{2\Pi}(a) \neq c_{2\Pi}(b)$.

Since all vertices in $J(m,n)$ have distinct color codes, c_2 is a locating-coloring in $J(m,n)$. Therefore, $\chi_L(J(m,n)) \leq n+2$.

□

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