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The e-open sets in Neutrosophic Hypersoft Topological Spaces and Application in Covid-19 Diagnosis using Normalized Hamming Distance

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Abstract. In this paper, we introduce a neutrosophic hypersoft e-open set which is the union of neutrosophic hypersoft δ-pre open sets and neutrosophic hypersoft δ-semi open sets in neutrosophic hypersoft topological spaces. Also, we discuss about the relations between neutrosophic hypersoft δ-pre open sets, neutrosophic hypersoft δ-semi open sets, neutrosophic hypersoft e-open sets and neutrosophic hypersoft e^*-open sets and their properties with the examples. Moreover, we investigate some of the basic properties of neutrosophic hypersoft e-interior and e-closure in a neutrosophic hypersoft topological space and proposed some examples for important results. Added to that, an application in Covid-19 diagnosis using normalized Hamming distance via neutrosophic hypersoft sets is discussed.

Key words and Phrases: neutrosophic hypersoft e-open sets, neutrosophic hypersoft e^*-open sets, neutrosophic hypersoft e-interior and neutrosophic hypersoft e-closure, normalized Hamming distance.

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1. Introduction

The real world decision making problems in medical diagnosis, engineering, economics, management, computer science, artificial intelligence, social sciences, environmental science and sociology contains more uncertain and inadequate data. The traditional mathematical methods cannot deal with these kind of problems due to the imprecise data. To deal the problems with uncertainty, Zadeh [29] introduced the fuzzy set in 1965 which contains the membership value in [0,1]. The topological structure on fuzzy set was developed by Chang [7] as fuzzy topological space. Then Atanassov [4] extended this idea as Intuitionistic fuzzy set in 1983 which includes both membership and non-membership values. Coker [8] introduced intuitionistic fuzzy set in a topology as intuitionistic fuzzy topological space. Nevertheless, it can deal only with the incomplete data but not with the inconsistent or indeterminate data. To overcome this issue, Smarandache [22, 23] introduced the neutrosophic set which contains membership, indeterminacy and non-membership values which are independent to each other. It can handle all kind of real life situations containing inconsistent, incomplete and indeterminate data. Salama and Alblowi [17] in 2012, developed neutrosophic topological space. A new mathematical tool, soft set theory was introduced by Molodstov [12] in 1999 to deal uncertainties in which a soft set is a collection of imprecise interpretations of an object. Soft set is a parameterized family of subsets where parameters are the properties, attributes or characteristics of the objects. The soft set theory have several applications in different fields such as decision making, optimization, forecasting, data analysis etc. Shabir and Naz [21] developed soft topological spaces.

Maji [11] combined the neutrosophic structure and the soft set concept to develop neutrosophic soft sets and the same was modified by Deli and Broumi [9]. Later neutrosophic soft topological spaces were presented by Bera [5]. Smarandache [24] extended the notion of a soft set to a hypersoft set and then to plithogenic set by replacing function with a multi-argument function described in the cartesian product with a different set of attributes. This new concept of hypersoft set is more flexible than the soft set and more suitable in the decision-making issues involving different kind of attributes. Saqlain et al. [18] defined the aggregate operators of neutrosophic hypersoft set. Ozturk and Yolcu [13] redefined the same and developed the neutrosophic hypersoft topological spaces. Ajay and Charisma [2] introduced fuzzy hypersoft topology, intuitionistic hypersoft topology and neutrosophic hypersoft topology. Ajay et al. [3] defined neutrosophic hypersoft semi-open sets and developed an application in multiattribute group decision making.

Saha [16] defined δ-open sets in fuzzy topological spaces. Vadivel et al. [25] introduced δ-open sets in neutrosophic topological spaces. In 2019, Acikgoz and Esenbel [1] defined neutrosophic soft δ-topology. The notion of e-open sets were introduced by Ekici [10] in a general topology, Seenivasan et al. [20] in fuzzy topological space, Chandrasekar et al. [6] in intuitionistic fuzzy topological space, Vadivel et al. [26, 27, 28] in neutrosophic topological spaces and Revathi et al. [14, 15] in neutrosophic soft topological spaces.

Saqlain et al. [19] studied distance and similarity measures for neutrosophic hypersoft set (NHSS) with construction of NHSS-TOPSIS and applications.

In this paper, we have developed the concept of neutrosophic hypersoft e-open sets in neutrosophic hypersoft topological spaces and also some of their basic properties with examples are specialized. Also, we discuss about neutrosophic hypersoft e-interior and e-closure in neutrosophic hypersoft topological spaces. An application in Covid-19 diagnosis using normalized Hamming distance involving neutrosophic hypersoft sets is also discussed.

2. Preliminaries

Definition 2.1. [17] Let $\mathfrak{M}$ be an initial universe. A neutrosophic set (briefly Ns) $\tilde{H}$ is an object having the form $\tilde{H} = \{\langle \mathfrak{m}, \mu_{\tilde{H}}(\mathfrak{m}), \sigma_{\tilde{H}}(\mathfrak{m}), \nu_{\tilde{H}}(\mathfrak{m})\rangle : \mathfrak{m} \in \mathfrak{M}\}$ where $\mu_{\tilde{H}} \to [0,1]$ denote the degree of membership function, $\sigma_{\tilde{H}} \to [0,1]$ denote the degree of indeterminacy function and $\nu_{\tilde{H}} \to [0,1]$ denote the degree of non-membership function respectively of each element $\mathfrak{m} \in \mathfrak{M}$ to the set $\tilde{H}$ and $0 \leq \mu_{\tilde{H}}(\mathfrak{m}) + \sigma_{\tilde{H}}(\mathfrak{m}) + \nu_{\tilde{H}}(\mathfrak{m}) \leq 3$ for each $\mathfrak{m} \in \mathfrak{M}$.

Definition 2.2. [12] Let $\mathfrak{M}$ be an initial universe, Q be a set of parameters and $\mathcal{P}(\mathfrak{M})$ be the power set of $\mathfrak{M}$. A pair $(\tilde{H}, Q)$ is called the a soft set over $\mathfrak{M}$ where $\tilde{H}$ is a mapping $\tilde{H} : Q \to \mathcal{P}(\mathfrak{M})$. In other words, the soft set is a parametrized family of subsets of the set $\mathfrak{M}$.

Definition 2.3. [9] Let $\mathfrak{M}$ be an initial universe, Q be a set of parameters. Let $\mathcal{P}(\mathfrak{M})$ denotes the set of all neutrosophic sets of $\mathfrak{M}$. Then a neutrosophic soft set $(\tilde{H}, Q)$ over $\mathfrak{M}$ (briefly N_sSs) is defined by
$(\tilde{H}, Q) = \{(q, \langle \mathfrak{m}, \mu_{\tilde{H}(q)}(\mathfrak{m}), \sigma_{\tilde{H}(q)}(\mathfrak{m}), \nu_{\tilde{H}(q)}(\mathfrak{m})\rangle : \mathfrak{m} \in \mathfrak{M}) : q \in Q\}$, where $\mu_{\tilde{H}(q)}(\mathfrak{m})$, $\sigma_{\tilde{H}(q)}(\mathfrak{m})$, $\nu_{\tilde{H}(q)}(\mathfrak{m}) \in [0,1]$ respectively called the degree of membership function, the degree of indeterminacy function and the degree of non-membership function of $\tilde{H}(q)$. Since the supremum of each μ, σ, ν is 1, the inequality $0 \leq \mu_{\tilde{H}(q)}(\mathfrak{m}) + \sigma_{\tilde{H}(q)}(\mathfrak{m}) + \nu_{\tilde{H}(q)}(\mathfrak{m}) \leq 3$ is obvious.

Definition 2.4. [24] Let $\mathfrak{M}$ be an initial universe and $\mathcal{P}(\mathfrak{M})$ be the power set of $\mathfrak{M}$. Consider $q_1, q_2, q_3, ..., q_n$ for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $Q_1,\ Q_2,\ ...,\ Q_n$ with $Q_i \cap Q_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$. Then the pair $(\tilde{H},\ Q_1 \times Q_2 \times ... \times Q_n)$ where $\tilde{H} : Q_1 \times Q_2 \times ... \times Q_n \to \mathcal{P}(\mathfrak{M})$ is called a hypersoft set over $\mathfrak{M}$.

Definition 2.5. [18] Let $\mathfrak{M}$ be an initial universal set and $Q_1, Q_2, ..., Q_n$ be pairwise disjoint sets of parameters. Let $\mathcal{P}(\mathfrak{M})$ be the set of all neutrosophic sets of $\mathfrak{M}$. Let E_i be the nonempty subset of the pair Q_i for each $i = 1, 2, ..., n$. A neutrosophic hypersoft set (briefly, N_sHSs) over $\mathfrak{M}$ is defined as the pair $(\tilde{H}, E_1 \times E_2 \times ... \times E_n)$ where $\tilde{H} : E_1 \times E_2 \times ... \times E_n \to \mathcal{P}(\mathfrak{M})$ and $\tilde{H}(E_1 \times E_2 \times ... \times E_n) = \{(q, \langle \mathfrak{m}, \mu_{\tilde{H}(q)}(\mathfrak{m}), \sigma_{\tilde{H}(q)}(\mathfrak{m}), \nu_{\tilde{H}(q)}(\mathfrak{m})\rangle : \mathfrak{m} \in \mathfrak{M}) : q \in E_1 \times E_2 \times ... \times E_n \subseteq Q_1 \times Q_2 \times ... \times Q_n\}$ where $\mu_{\tilde{H}(q)}(\mathfrak{m})$ is the membership value of truthiness, $\sigma_{\tilde{H}(q)}(\mathfrak{m})$ is the membership value of indeterminacy and $\nu_{\tilde{H}(q)}(\mathfrak{m})$ is the membership value of falsity such that $\mu_{\tilde{H}(q)}(\mathfrak{m}), \sigma_{\tilde{H}(q)}(\mathfrak{m}), \nu_{\tilde{H}(q)}(\mathfrak{m}) \in [0,1]$. Also, $0 \leq \mu_{\tilde{H}(q)}(\mathfrak{m}) + \sigma_{\tilde{H}(q)}(\mathfrak{m}) + \nu_{\tilde{H}(q)}(\mathfrak{m}) \leq 3$.

Definition 2.6. [18] Let $\mathfrak{M}$ be an universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be two N_sHSs's over $\mathfrak{M}$. Then $(\tilde{H}, \wedge_1)$ is the neutrosophic hypersoft subset of $(\tilde{G}, \wedge_2)$ if $\mu_{\tilde{H}(q)}(\mathfrak{m}) \leq \mu_{\tilde{G}(q)}(\mathfrak{m}), \sigma_{\tilde{H}(q)}(\mathfrak{m}) \leq \sigma_{\tilde{G}(q)}(\mathfrak{m}), \nu_{\tilde{H}(q)}(\mathfrak{m}) \leq \nu_{\tilde{G}(q)}(\mathfrak{m})$. It is denoted by $(\tilde{H}, \wedge_1) \subseteq (\tilde{G}, \wedge_2)$.

Definition 2.7. [18] Let $\mathfrak{M}$ be an universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be N_sHSs's over $\mathfrak{M}$. $(\tilde{H}, \wedge_1)$ is equal to $(\tilde{G}, \wedge_1)$ if $\mu_{\tilde{H}(q)}(\mathfrak{m}) = \mu_{\tilde{G}(q)}(\mathfrak{m}), \sigma_{\tilde{H}(q)}(\mathfrak{m}) = \sigma_{\tilde{G}(q)}(\mathfrak{m})$, $\nu_{\tilde{H}(q)}(\mathfrak{m}) = \nu_{\tilde{G}(q)}(\mathfrak{m})$.

Definition 2.8. [13] Let $\mathfrak{M}$ be an universal set and $((\tilde{H}, \wedge)$ be N_sHSs over $\mathfrak{M}$. $((\tilde{H}, \wedge)^c$ is the complement of N_sHSs of $((\tilde{H}, \wedge)$ if $\mu^c_{\tilde{H}(q)}(\mathfrak{m}) = \nu_{\tilde{H}(q)}(\mathfrak{m})$, $\sigma^c_{\tilde{H}(q)}(\mathfrak{m}) = 1 - \sigma_{\tilde{H}(q)}(\mathfrak{m})$, $\nu^c_{\tilde{H}(q)}(\mathfrak{m}) = \mu_{\tilde{H}(q)}(\mathfrak{m})$ where $\forall q \in \wedge$ and $\forall \mathfrak{m} \in \mathfrak{M}$.

It is clear that $(((\tilde{H}, \wedge)^c)^c = ((\tilde{H}, \wedge)$.

Definition 2.9. [13] A N_sHSs $((\tilde{H}, \wedge)$ over the universe set $\mathfrak{M}$ is said to be null neutrosophic hypersoft set if $\mu_{\tilde{H}(q)}(\mathfrak{m}) = 0,\ \sigma_{\tilde{H}(q)}(\mathfrak{m}) = 0,\ \nu_{\tilde{H}(q)}(\mathfrak{m}) = 1\ \forall q \in \wedge$ and $\mathfrak{m} \in \mathfrak{M}$. It is denoted by $\tilde{0}_{(\mathfrak{M},Q)}$.

A N_sHSs $(\tilde{G}, \wedge)$ over the universal set $\mathfrak{M}$ is said to be absolute neutrosophic hypersoft set if $\mu_{\tilde{H}(q)}(\mathfrak{m}) = 1, \sigma_{\tilde{H}(q)}(\mathfrak{m}) = 1, \nu_{\tilde{H}(q)}(\mathfrak{m}) = 0\ \forall q \in \wedge$ and $\mathfrak{m} \in \mathfrak{M}$. It is denoted by $\tilde{1}_{(\mathfrak{M},Q)}$.

Clearly, $\tilde{0}^c_{(\mathfrak{M},Q)} = \tilde{1}_{(\mathfrak{M},Q)}$ and $\tilde{1}^c_{(\mathfrak{M},Q)} = \tilde{0}_{(\mathfrak{M},Q)}$.

Definition 2.10. [13] Let $\mathfrak{M}$ be the universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be N_sHSs's over $\mathfrak{M}$. Extended union $(\tilde{H}, \wedge_1) \cup (\tilde{G}, \wedge_2)$ is defined as

$$\mu((\tilde{H}, \wedge_1) \cup (\tilde{G}, \wedge_2)) = \begin{cases} \mu_{\tilde{H}(q)}(\mathfrak{m}) & \text{if } q \in \wedge_1 - \wedge_2 \\ \mu_{\tilde{G}(q)}(\mathfrak{m}) & \text{if } q \in \wedge_2 - \wedge_1 \\ max\{\mu_{\tilde{H}(q)}(\mathfrak{m}), \mu_{\tilde{G}(q)}(\mathfrak{m})\} & \text{if } q \in \wedge_1 \cap \wedge_2 \end{cases}$$

$$\sigma((\tilde{H}, \wedge_1) \cup (\tilde{G}, \wedge_2)) = \begin{cases} \sigma_{\tilde{H}(q)}(\mathfrak{m}) & \text{if} q \in \wedge_1 - \wedge_2 \\ \sigma_{\tilde{G}(q)}(\mathfrak{m}) & \text{if} q \in \wedge_2 - \wedge_1 \\ max\{\sigma_{\tilde{H}(q)}(\mathfrak{m}), \sigma_{\tilde{G}(q)}(\mathfrak{m})\} & \text{if } q \in \wedge_1 \cap \wedge_2 \end{cases}$$

$$\nu((\tilde{H}, \wedge_1) \cup (\tilde{G}, \wedge_2)) = \begin{cases} \nu_{\tilde{H}(q)}(\mathfrak{m}) & ifq \in \wedge_1 - \wedge_2 \\ \nu_{\tilde{G}(q)}(\mathfrak{m}) & ifq \in \wedge_2 - \wedge_1 \\ min\{\nu_{\tilde{H}(q)}(\mathfrak{m}), \nu_{\tilde{G}(q)}(\mathfrak{m})\} & ifq \in \wedge_1 \cap \wedge_2 \end{cases}$$

Definition 2.11. [13] Let $\mathfrak{M}$ be the universal set and $(\tilde{H}, \wedge_1)$ and $(\tilde{G}, \wedge_2)$ be N_sHS's over $\mathfrak{M}$. Extended intersection $(\tilde{H}, \wedge_1) \cap (\tilde{G}, \wedge_2)$ is defined as

$$\mu((\tilde{H},\wedge_1)\cap(\tilde{G},\wedge_2))=\begin{cases}\mu_{\tilde{H}(q)}(\mathfrak{m}) & if q\in\wedge_1-\wedge_2\\ \mu_{\tilde{G}(q)}(\mathfrak{m}) & if q\in\wedge_2-\wedge_1\\ min\{\mu_{\tilde{H}(q)}(\mathfrak{m}),\mu_{\tilde{G}(q)}(\mathfrak{m})\} & if q\in\wedge_1\cap\wedge_2\end{cases}$$

$$\sigma((\tilde{H},\wedge_1)\cap(\tilde{G},\wedge_2))=\begin{cases}\sigma_{\tilde{H}(q)}(\mathfrak{m}) & if q\in\wedge_1-\wedge_2\\ \sigma_{\tilde{G}(q)}(\mathfrak{m}) & if q\in\wedge_2-\wedge_1\\ min\{\sigma_{\tilde{H}(q)}(\mathfrak{m}),\sigma_{\tilde{G}(q)}(\mathfrak{m})\} & if q\in\wedge_1\cap\wedge_2\end{cases}$$

$$\nu((\tilde{H},\wedge_1)\cap(\tilde{G},\wedge_2))=\begin{cases}\nu_{\tilde{H}(q)}(\mathfrak{m}) & if q\in\wedge_1-\wedge_2\\ \nu_{\tilde{G}(q)}(\mathfrak{m}) & if q\in\wedge_2-\wedge_1\\ max\{\nu_{\tilde{H}(q)}(\mathfrak{m}),\nu_{\tilde{G}(q)}(\mathfrak{m})\} & if q\in\wedge_1\cap\wedge_2\end{cases}$$

Definition 2.12. [13] Let $\{(\tilde{H}_i,\wedge_i)|i\in I\}$ be a family of N_sHSs's over the universe set $\mathfrak{M}$. Then

$\bigcup_{i\in I}(\tilde{H}_i,\wedge_i) = \{\langle\mathfrak{m}, sup[\mu_{\tilde{H}_i(q)}(\mathfrak{m})]_{i\in I}, sup[\sigma_{\tilde{H}_i(q)}(\mathfrak{m})]_{i\in I}, inf[\nu_{\tilde{H}_i(q)}(\mathfrak{m})]_{i\in I}\rangle : \mathfrak{m}\in\mathfrak{M}\}$

$\bigcap_{i\in I}(\tilde{H}_i,\wedge_i) = \{\langle\mathfrak{m}, inf[\mu_{\tilde{H}_i(q)}(\mathfrak{m})]_{i\in I}, inf[\sigma_{\tilde{H}_i(q)}(\mathfrak{m})]_{i\in I}, sup[\nu_{\tilde{H}_i(q)}(\mathfrak{m})]_{i\in I}\rangle : \mathfrak{m}\in\mathfrak{M}\}$.

Definition 2.13. [13] Let (Y,Q) be the family of all N_sHSs's over the universe set $\mathfrak{M}$ and $\tau\subseteq N_sHSs(Y,Q)$. Then τ is said to be a neutrosophic hypersoft topology (briefly, N_sHSt) on $\mathfrak{M}$ if

(i) $\tilde{0}_{(\mathfrak{M},Q)}$ and $\tilde{1}_{(\mathfrak{M},Q)}$ belongs to τ
(ii) the union of any number of N_sHSs's in τ belongs to τ
(iii) the intersection of finite number of N_sHSs's in τ belongs to τ.

Then $(\mathfrak{M},Q,\tau)$ is called a neutrosophic hypersoft toplogical space (briefly, N_sHSts) over $\mathfrak{M}$. Each member of τ is said to be neutrosophic hypersoft open set (briefly, N_sHSos). A $N_sHSs((\tilde{H},\wedge)$ is called a neutrosophic hypersoft closed set (briefly, N_sHScs) if its complement $((\tilde{H},\wedge)^c$ is N_sHSos.

The intuitionisic hypersoft topological space and fuzzy topological space are defined in [2].

Definition 2.14. [13] Let $(\mathfrak{M},Q,\tau)$ be a N_sHSts over $\mathfrak{M}$ and $((\tilde{H},\wedge)\in N_sHSs(Y, Q)$ be a N_sHSs. Then, the neutrosophic hypersoft interior (briefly, N_sHSint) of $((\tilde{H},\wedge)$ is defined as $N_sHSint((\tilde{H},\wedge)=\cup\{(\tilde{G},\wedge):(\tilde{G},\wedge)\subseteq((\tilde{H},\wedge)$ where $(\tilde{G},\wedge)$ is $N_sHSos\}$.

Definition 2.15. [13] Let $(\mathfrak{M},Q,\tau)$ be a N_sHSts over $\mathfrak{M}$ and $((\tilde{H},\wedge)\in N_sHSs(Y, Q)$ be a N_sHSs. Then, the neutrosophic hypersoft closure (briefly, N_sHScl) of $((\tilde{H},\wedge)$ is defined as $N_sHScl((\tilde{H},\wedge)=\cap\{(\tilde{G},\wedge):(\tilde{G},\wedge)\supseteq((\tilde{H},\wedge)$ where $(\tilde{G},\wedge)$ is $N_sHScs\}$.

Definition 2.16. [3] Let $(\mathfrak{M},Q,\tau)$ be a N_sHSts over $\mathfrak{M}$ and $((\tilde{H},\wedge)\in N_sHSs(\mathfrak{M}, Q)$ be a N_sHSs. Then, $((\tilde{H},\wedge)$ is called the neutrosophic hypersoft semiopen set (briefly, N_sHSSos) if $((\tilde{H},\wedge)\subseteq N_sHScl(int((\tilde{H},\wedge))$.

A $N_sHSs((\tilde{H},\wedge)$ is called a neutrosophic hypersoft semiclosed set (briefly, $N_sHS\mathcal{S}cs$) if its complement $((\tilde{H},\wedge)^c$ is a $N_sHS\mathcal{S}os$.

Definition 2.17. [19] Consider two N_sHSs's $(\tilde{H},\wedge_1)$ and $(\tilde{G},\wedge_2)$ over $\mathfrak{M}$. The normalized Hamming distance for these two sets are given by $d_{NH}((\tilde{H},\wedge_1),(\tilde{G},\wedge_2)) = \frac{1}{3n}\sum_{i=1}^{n}|\mu_H^i-\mu_G^i|+|\sigma_H^i-\sigma_G^i|+|\nu_H^i-\nu_G^i|$.

3. Neutrosophic hypersoft δ-open sets in N_sHSts

Definition 3.1. Let $(\mathfrak{M},Q,\tau)$ be a N_sHSts over $\mathfrak{M}$. A N_sHSs $((\tilde{H},\wedge)$ is said to be a neutrosophic hypersoft regular open set (briefly, N_sHSros) if $((\tilde{H},\wedge) = N_sHSint(N_sHScl((\tilde{H},\wedge))$. The complement of N_sHSros is called a neutrosophic hypersoft regular closed set (briefly, N_sHSrcs) in $\mathfrak{M}$.

Definition 3.2. Let $(\mathfrak{M},Q,\tau)$ be a N_sHSts over $\mathfrak{M}$ and $((\tilde{H},\wedge)$ be a N_sHSs on $\mathfrak{M}$. Then the neutrosophic hypersoft

(i) δ-interior (briefly, N_sHSint) of $((\tilde{H},\wedge)$ is defined by

$$N_sHS\delta int((\tilde{H},\wedge)=\bigcup\{(\tilde{G},\wedge):(\tilde{G},\wedge)\subseteq((\tilde{H},\wedge)$$

and $(\tilde{G},\wedge)$ is a N_sHSros in $\mathfrak{M}\}$

(ii) δ-closure (briefly, N_sHScl) of $((\tilde{H},\wedge)$ is defined by

$$N_sHS\delta cl((\tilde{H},\wedge)=\bigcap\{(\tilde{G},\wedge):(\tilde{G},\wedge)\supseteq((\tilde{H},\wedge)$$

and $(\tilde{G},\wedge)$ is a N_sHSrcs in $\mathfrak{M}\}$

Definition 3.3. Let $(\mathfrak{M},Q,\tau)$ be a N_sHSts over $\mathfrak{M}$. A N_sHSs $((\tilde{H},\wedge)$ is said to be a neutrosophic hypersoft

(i) semi-regular if $((\tilde{H},\wedge)$ is both $N_sHS\mathcal{S}os$ and $N_sHS\mathcal{S}cs$.
(ii) pre open set (briefly, $N_sHS\mathcal{P}os$) if $((\tilde{H},\wedge)\subseteq N_sHSint(N_sHScl((\tilde{H},\wedge)$
(iii) δ-open set (briefly, $N_sHS\delta os$) if $((\tilde{H},\wedge)=N_sHS\delta int((\tilde{H},\wedge)$
(iv) δ-pre open set (briefly, $N_sHS\delta\mathcal{P}os$) if $((\tilde{H},\wedge)\subseteq N_sHSint(N_sHS\delta cl((\tilde{H},\wedge))$
(v) δ-semi open set (briefly, $N_sHS\delta\mathcal{S}os$) if $((\tilde{H},\wedge)\subseteq N_sHScl(N_sHS\delta int((\tilde{H},\wedge))$

The complement of $N_sHS\delta os$ (resp. $N_sHS\mathcal{P}os$, $N_sHS\delta\mathcal{P}os$ & $N_sHS\delta\mathcal{S}os$) is called a $N_sHS\delta$ (resp. N_sHS pre, $N_sHS\delta$ pre & $N_sHS\delta$ semi) closed set (briefly, $N_sHS\delta cs$ (resp. $N_sHS\mathcal{P}cs$, $N_sHS\delta\mathcal{P}cs$ & $N_sHS\delta\mathcal{S}cs$)) in $\mathfrak{M}$.

The family of all $N_sHS\delta os$ (resp. $N_sHS\delta cs$, N_sHSros, N_sHSrcs, $N_sHS\mathcal{P}os$, $N_sHS\mathcal{P}cs$ $N_sHS\delta\mathcal{P}os$, $N_sHS\delta\mathcal{P}cs$, $N_sHS\delta\mathcal{S}os$ & $N_sHS\delta\mathcal{S}cs$) of $\mathfrak{M}$ is denoted by $N_sHS\delta OS(\mathfrak{M})$ (resp. $N_sHS\delta CS(\mathfrak{M})$, $N_sHSrOS(\mathfrak{M})$, $N_sHSrOS(\mathfrak{M})$, $N_sHS\mathcal{P}OS(\mathfrak{M})$, $N_sHS\mathcal{P}CS(\mathfrak{M})$, $N_sHS\delta\mathcal{P}OS(\mathfrak{M})$, $N_sHS\delta\mathcal{P}CS(\mathfrak{M})$, $N_sHS\delta\mathcal{S}OS(\mathfrak{M})$ & $N_sHS\delta\mathcal{S}CS(\mathfrak{M})$).

Definition 3.4. Let $(\mathfrak{M},Q,\tau)$ be a N_sHSts over $\mathfrak{M}$ and $((\tilde{H},\wedge)$ be a N_sHSs on $\mathfrak{M}$. Then the neutrosophic hypersoft

(i) δ-pre (resp. δ-semi) interior (briefly, $N_sHS\delta\mathcal{P}int$ (resp. $N_sHS\delta\mathcal{S}int$)) of $((\tilde{H},\wedge)$ is defined by

$$N_sHS\delta\mathcal{P}int((\tilde{H},\wedge) = \bigcup\{(\tilde{G},\wedge) : (\tilde{G},\wedge) \subseteq ((\tilde{H},\wedge)$$

and $(\tilde{G},\wedge)$ is a $N_sHS\ \delta\mathcal{P}os$ (resp. $N_sHS\delta\mathcal{S}os$) in $\mathfrak{M}\}$

(ii) δ-pre (resp. δ-semi) closure (briefly, $N_sHS\delta\mathcal{P}cl$ (resp. $N_sHS\delta\mathcal{S}cl$)) of $((\tilde{H},\wedge)$ is defined by

$$N_sHS\delta\mathcal{P}cl((\tilde{H},\wedge) = \bigcap\{(\tilde{G},\wedge) : (\tilde{G},\wedge) \supseteq ((\tilde{H},\wedge)$$

and $(\tilde{G},\wedge)$ is a $N_sHS\delta\mathcal{P}\ cs$ (resp. $N_sHS\delta\mathcal{S}cs$) in $\mathfrak{M}\}$

Definition 3.5. Let $(\mathfrak{M}, Q, \tau_I)$ be an intuitionistic hypersoft topological space (briefly, $IHSts$) over $\mathfrak{M}$. An intuitionistic hypersoft set (briefly, $IHSs$) $((\tilde{H},\wedge)$ is said to be an intuitionistic hypersoft regular open set (briefly, $IHSros$) if $((\tilde{H},\wedge) = IHSint(IHScl((\tilde{H},\wedge))$. The complement of $IHSros$ is called an intuitionistic hypersoft regular closed set (briefly, $IHSrcs$) in $\mathfrak{M}$.

Definition 3.6. Let $(\mathfrak{M}, Q, \tau_I)$ be an $IHSts$ over $\mathfrak{M}$ and $((\tilde{H},\wedge)$ be an $IHSs$ on $\mathfrak{M}$. Then the intuitionistic hypersoft (briefly, IHS)

(i) δ-interior (briefly, $IHSint$) of $((\tilde{H},\wedge)$ is defined by

$$IHS\delta int((\tilde{H},\wedge) = \bigcup\{(\tilde{G},\wedge) : (\tilde{G},\wedge) \subseteq ((\tilde{H},\wedge)$$

and $(\tilde{G},\wedge)$ is a $IHSros$ in $\mathfrak{M}\}$

(ii) δ-closure (briefly, $IHScl$) of $((\tilde{H},\wedge)$ is defined by

$$IHS\delta cl((\tilde{H},\wedge) = \bigcap\{(\tilde{G},\wedge) : (\tilde{G},\wedge) \supseteq ((\tilde{H},\wedge)$$

and $(\tilde{G},\wedge)$ is a $IHSrcs$ in $\mathfrak{M}\}$

Definition 3.7. Let $(\mathfrak{M}, Q, \tau_I)$ be an $IHSts$ over $\mathfrak{M}$. An $IHSs$ $((\tilde{H},\wedge)$ is said to be an intuitionistic hypersoft

(i) semi-regular if $((\tilde{H},\wedge)$ is both $IHS\mathcal{S}os$ and $IHS\mathcal{S}cs$.
(ii) pre open set (briefly, $IHS\mathcal{P}os$) if $((\tilde{H},\wedge) \subseteq IHSint(IHScl((\tilde{H},\wedge)$
(iii) δ-open set (briefly, $IHS\delta os$) if $((\tilde{H},\wedge) = IHS\delta int((\tilde{H},\wedge)$
(iv) δ-pre open set (briefly, $IHS\delta\mathcal{P}os$) if $((\tilde{H},\wedge) \subseteq IHSint(IHS\delta cl((\tilde{H},\wedge))$
(v) δ-semi open set (briefly, $IHS\delta\mathcal{S}os$) if $((\tilde{H},\wedge) \subseteq IHScl(IHS\delta int((\tilde{H},\wedge))$

The complement of $IHS\delta os$ (resp. $IHS\mathcal{P}os$, $IHS\delta\mathcal{P}os$ & $IHS\delta\mathcal{S}os$) is called a $IHS\delta$ (resp. IHS pre, $IHS\delta$ pre & $IHS\delta$ semi) closed set (briefly, $IHS\delta cs$ (resp. $IHS\mathcal{P}cs$, $IHS\delta\mathcal{P}cs$ & $IHS\delta\mathcal{S}cs$)) in $\mathfrak{M}$.

The family of all $IHS\delta os$ (resp. $IHS\delta cs$, $IHSros$, $IHSrcs$, $IHS\mathcal{P}os$, $IHS\mathcal{P}\ cs$ $IHS\delta\mathcal{P}os$, $IHS\delta\mathcal{P}cs$, $IHS\delta\mathcal{S}os$ & $IHS\delta\mathcal{S}cs$) of $\mathfrak{M}$ is denoted by $IHS\delta OS(\mathfrak{M})$ (resp. $IHS\delta CS(\mathfrak{M})$, $IHSrOS(\mathfrak{M})$, $IHSrOS(\mathfrak{M})$, $IHS\mathcal{P}OS(\mathfrak{M})$, $IHS\mathcal{P}CS(\mathfrak{M})$, $IHS\delta\mathcal{P}OS(\mathfrak{M})$, $IHS\delta\mathcal{P}CS(\mathfrak{M})$, $IHS\delta\mathcal{S}OS(\mathfrak{M})$ & $IHS\delta\mathcal{S}CS(\mathfrak{M})$).

Definition 3.8. Let $(\mathfrak{M}, Q, \tau_I)$ be a $IHSts$ over $\mathfrak{M}$ and $((\tilde{H}, \wedge)$ be a $IHSs$ on $\mathfrak{M}$. Then the intuitionistic hypersoft

(i) δ-pre (resp. δ-semi) interior (briefly, $IHS\delta\mathcal{P}int$ (resp. $IHS\delta\mathcal{S}int$)) of $((\tilde{H}, \wedge)$ is defined by

$$IHS\delta\mathcal{P}int((\tilde{H}, \wedge) = \bigcup\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \subseteq ((\tilde{H}, \wedge)$$

and $(\tilde{G}, \wedge)$ is a $IHS\delta\mathcal{P}os$ (resp. $IHS\delta\mathcal{S}os$) in $\mathfrak{M}\}$

(ii) δ-pre (resp. δ-semi) closure (briefly, $IHS\delta\mathcal{P}cl$ (resp. $IHS\delta\mathcal{S}cl$)) of $((\tilde{H}, \wedge)$ is defined by

$$IHS\delta\mathcal{P}cl((\tilde{H}, \wedge) = \bigcap\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \supseteq ((\tilde{H}, \wedge)$$

and $(\tilde{G}, \wedge)$ is a $IHS\delta\mathcal{P}cs$ (resp. $IHS\delta\mathcal{S}cs$) in $\mathfrak{M}\}$

Definition 3.9. Let $(\mathfrak{M}, Q, \tau_F)$ be a fuzzy hypersoft topological space (briefly, $FHSts$) over $\mathfrak{M}$. An fuzzy hypersoft set (briefly, $FHSs$) $((\tilde{H}, \wedge)$ is said to be a fuzzy hypersoft regular open set (briefly, $FHSros$) if $((\tilde{H}, \wedge) = FHSint(FHScl((\tilde{H}, \wedge))$. The complement of $FHSros$ is called a fuzzy hypersoft regular closed set (briefly, $FHSrcs$) in $\mathfrak{M}$.

Definition 3.10. Let $(\mathfrak{M}, Q, \tau_F)$ be a $FHSts$ over $\mathfrak{M}$ and $((\tilde{H}, \wedge)$ be a $FHSs$ on $\mathfrak{M}$. Then the fuzzy hypersoft (briefly, FHS)

(i) δ-interior (briefly, $FHSint$) of $((\tilde{H}, \wedge)$ is defined by
$FHS\delta int((\tilde{H}, \wedge) = \bigcup\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \subseteq ((\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a $FHSros$ in $\mathfrak{M}\}$

(ii) δ-closure (briefly, $FHScl$) of $((\tilde{H}, \wedge)$ is defined by
$FHS\delta cl((\tilde{H}, \wedge) = \bigcap\{(\tilde{G}, \wedge) : (\tilde{G}, \wedge) \supseteq ((\tilde{H}, \wedge)$ and $(\tilde{G}, \wedge)$ is a $FHSrcs$ in $\mathfrak{M}\}$

Definition 3.11. Let $(\mathfrak{M}, Q, \tau_F)$ be a $FHSts$ over $\mathfrak{M}$. An $FHSs$ $((\tilde{H}, \wedge)$ is said to be a fuzzy hypersoft

(i) semi-regular if $((\tilde{H}, \wedge)$ is both $FHS\mathcal{S}os$ and $FHS\mathcal{S}cs$.
(ii) pre open set (briefly, $FHS\mathcal{P}os$) if $((\tilde{H}, \wedge) \subseteq FHSint(FHScl((\tilde{H}, \wedge)$
(iii) δ-open set (briefly, $FHS\delta os$) if $((\tilde{H}, \wedge) = FHS\delta int((\tilde{H}, \wedge)$
(iv) δ-pre open set (briefly, $FHS\delta\mathcal{P}os$) if $((\tilde{H}, \wedge) \subseteq FHSint(FHS\delta cl((\tilde{H}, \wedge))$
(v) δ-semi open set (briefly, $FHS\delta\mathcal{S}os$) if $((\tilde{H}, \wedge) \subseteq FHScl(FHS\delta int((\tilde{H}, \wedge))$

The complement of $FHS\delta os$ (resp. $FHS\mathcal{P}os$, $FHS\delta\mathcal{P}os$ & $FHS\delta\mathcal{S}os$) is called a $FHS\delta$ (resp. FHS pre, $FHS\delta$ pre & $FHS\delta$ semi) closed set (briefly, $FHS\delta cs$ (resp. $FHS\mathcal{P}cs$, $FHS\delta\mathcal{P}cs$ & $FHS\delta\mathcal{S}cs$)) in $\mathfrak{M}$.

The family of all $FHS\delta os$ (resp. $FHS\delta cs$, $FHSros$, $FHSrcs$, $FHS\mathcal{P}os$, $FHS\mathcal{P}cs$ $FHS\delta\mathcal{P}os$, $FHS\delta\mathcal{P}cs$, $FHS\delta\mathcal{S}os$ & $FHS\delta\mathcal{S}cs$) of $\mathfrak{M}$ is denoted by $FHS\delta OS(\mathfrak{M})$ (resp. $FHS\delta CS(\mathfrak{M})$, $FHSrOS(\mathfrak{M})$, $FHSrOS(\mathfrak{M})$, $FHS\mathcal{P}OS(\mathfrak{M})$, $FHS\mathcal{P}CS(\mathfrak{M})$, $FHS\delta\mathcal{P}OS(\mathfrak{M})$, $FHS\delta\mathcal{P}CS(\mathfrak{M})$, $FHS\delta\mathcal{S}OS(\mathfrak{M})$ & $FHS\delta\mathcal{S}CS(\mathfrak{M})$).

Definition 3.12. Let $(\mathfrak{M}, Q, \tau_F)$ be a $FHSts$ over $\mathfrak{M}$ and $((\tilde{H}, \wedge)$ be a $FHSs$ on $\mathfrak{M}$. Then the fuzzy hypersoft

(i) δ-pre (resp. δ-semi) interior (briefly, $FHS\delta\mathcal{P}int$ (resp. $FHS\delta\mathcal{S}int$)) of $((\tilde{H},\wedge)$ is defined by $FHS\delta\mathcal{P}int((\tilde{H},\wedge) = \bigcup\{(\tilde{G},\wedge) : (\tilde{G},\wedge) \subseteq ((\tilde{H},\wedge)$ and $(\tilde{G},\wedge)$ is a $FHS\delta\mathcal{P}os$ (resp. $FHS\delta\mathcal{S}os$) in $\mathfrak{M}\}$

(ii) δ-pre (resp. δ-semi) closure (briefly, $FHS\delta\mathcal{P}cl$ (resp. $FHS\delta\mathcal{S}cl$)) of $((\tilde{H},\wedge)$ is defined by $FHS\delta\mathcal{P}cl((\tilde{H},\wedge) = \bigcap\{(\tilde{G},\wedge) : (\tilde{G},\wedge) \supseteq ((\tilde{H},\wedge)$ and $(\tilde{G},\wedge)$ is a $FHS\delta\mathcal{P}cs$ (resp. $FHS\delta\mathcal{S}cs$) in $\mathfrak{M}\}$

Example 3.13. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ be a $NsHS$ initial universe and the attributes be Q_1, Q_2. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\},\ Q_2 = \{b_1, b_2\}.$$

Suppose that

$$E_1 = \{a_1, a_2\}, E_2 = \{b_1\}$$
$$D_1 = \{a_1\}, D_2 = \{b_1, b_2\}$$

are subsets of Q_i for each $i = 1, 2$. Then the N_sHSs $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_3)$, $(\tilde{H}_4, \wedge_3)$ over the universe $\mathfrak{M}$ are as follows.

$$(\tilde{H}_1, \wedge_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8,0.8,0.2}, \frac{\mathfrak{m}_2}{0.6,0.8,0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7,0.8,0.3}, \frac{\mathfrak{m}_2}{0.5,0.5,0.2}\}\rangle \end{Bmatrix}$$

$$(\tilde{H}_2, \wedge_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2,0.4,0.6}, \frac{\mathfrak{m}_2}{0.3,0.5,0.6}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5,0.5,0.4}, \frac{\mathfrak{m}_2}{0.4,0.5,0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{H}_3, \wedge_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2,0.4,0.6}, \frac{\mathfrak{m}_2}{0.3,0.5,0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7,0.8,0.3}, \frac{\mathfrak{m}_2}{0.5,0.5,0.2}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5,0.5,0.4}, \frac{\mathfrak{m}_2}{0.4,0.5,0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{H}_4, \wedge_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8,0.8,0.2}, \frac{\mathfrak{m}_2}{0.6,0.8,0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7,0.8,0.3}, \frac{\mathfrak{m}_2}{0.5,0.5,0.2}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5,0.5,0.4}, \frac{\mathfrak{m}_2}{0.4,0.5,0.5}\}\rangle \end{Bmatrix}$$

Then $\tau = \{0_{(\mathfrak{M},Q)}, 1_{(\mathfrak{M},Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_3), (\tilde{H}_4, \wedge_3)\}$ is a N_sHSts.

Remark 3.14. From N_sHSt we can deduce $IHSt$ and $FHSt$. $IHSt$ is obtained by considering the membership values and non membership values whereas $FHSt$ is obtained by considering only membership values. For example,

Example 3.15. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ be an IHS initial universe and the attributes be Q_1, Q_2. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\},\ Q_2 = \{b_1, b_2\}$$

Suppose that

$$E_1 = \{a_1, a_2\}, E_2 = \{b_1\}$$
$$D_1 = \{a_1\}, D_2 = \{b_1, b_2\}$$

are subsets of Q_i for each $i = 1, 2$. Then the $IHSs$ $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_3)$, $(\tilde{H}_4, \wedge_4)$ over the universe $\mathfrak{M}$ are as follows.

$$(\tilde{H}_1, \wedge_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8,0.2}, \frac{\mathfrak{m}_2}{0.6,0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7,0.3}, \frac{\mathfrak{m}_2}{0.5,0.2}\}\rangle \end{Bmatrix}$$

$$(\tilde{H}_2, \wedge_2) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2,0.6}, \frac{\mathfrak{m}_2}{0.3,0.6}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5,0.4}, \frac{\mathfrak{m}_2}{0.4,0.5}\} \rangle \end{matrix} \right\}$$

$$(\tilde{H}_3, \wedge_3) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2,0.6}, \frac{\mathfrak{m}_2}{0.3,0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7,0.3}, \frac{\mathfrak{m}_2}{0.5,0.2}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5,0.4}, \frac{\mathfrak{m}_2}{0.4,0.5}\} \rangle \end{matrix} \right\}$$

$$(\tilde{H}_4, \wedge_3) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8,0.2}, \frac{\mathfrak{m}_2}{0.6,0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7,0.3}, \frac{\mathfrak{m}_2}{0.5,0.2}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5,0.4}, \frac{\mathfrak{m}_2}{0.4,0.5}\} \rangle \end{matrix} \right\}$$

Then $\tau = \{0_{(\mathfrak{M},Q)}, 1_{(\mathfrak{M},Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_3), (\tilde{H}_4, \wedge_3)\}$ is a $IHSts$.

Example 3.16. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ be an FHS initial universe and the attributes be Q_1, Q_2. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\},\ Q_2 = \{b_1, b_2\}$$

Suppose that

$$E_1 = \{a_1, a_2\}, E_2 = \{b_1\}$$
$$D_1 = \{a_1\}, D_2 = \{b_1, b_2\}$$

are subsets of Q_i for each $i = 1, 2$. Then the $FHSs$ $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_3)$, $(\tilde{H}_4, \wedge_4)$ over the universe $\mathfrak{M}$ are as follows.

$$(\tilde{H}_1, \wedge_1) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{matrix} \right\}$$

$$(\tilde{H}_2, \wedge_2) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \end{matrix} \right\}$$

$$(\tilde{H}_3, \wedge_3) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \end{matrix} \right\}$$

$$(\tilde{H}_4, \wedge_3) = \left\{ \begin{matrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \end{matrix} \right\}$$

Then $\tau = \{0_{(\mathfrak{M},Q)}, 1_{(\mathfrak{M},Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_3), (\tilde{H}_4, \wedge_3)\}$ is a $FHSts$.

4. NEUTROSOPHIC HYPERSOFT e-OPEN SETS IN N_sHSts

Definition 4.1. A set $(\tilde{H}, \wedge)$ is said to be a neutrosophic hypersoft

(i) e-open set (briefly, N_sHSeos) if $(\tilde{H}, \wedge) \subseteq N_sHScl\big(N_sHS\delta int(\tilde{H}, \wedge)\big) \cup N_sHS\ int\big(N_sHS\delta cl(\tilde{H}, \wedge)\big)$.

(ii) e^*-open set (briefly, N_sHSe^*os) if $(\tilde{H}, \wedge) \subseteq N_sHScl\big(N_sHSint(N_sHS\delta cl\ (\tilde{H}, \wedge))\big)$.

The complement of a N_sHSe-open set (resp. N_sHSe^*os) is called a neutrosophic hypersoft e- (resp. e^*) closed set (briefly, N_sHSecs (resp. N_sHSe^*cs)) in $\mathfrak{M}$.

The family of all N_sHSeos (resp. N_sHSecs N_sHSe^*os & N_sHSe^*cs) of $\mathfrak{M}$ is denoted by $N_sHSeOS(\mathfrak{M})$ (resp. $N_sHSeCS(\mathfrak{M})$, $N_sHSe^*OS(\mathfrak{M})$ & N_sHSe^*CS $(\mathfrak{M})$).

Definition 4.2. A set $(\tilde{H}, \wedge)$ is said to be a neutrosophic hypersoft

(i) e interior (briefly, $N_sHSeint(\tilde{H}, \wedge)$ is defined by
$N_sHSeint(\tilde{H}, \wedge) = \bigcup\{(\tilde{L}, \wedge) : (\tilde{L}, \wedge) \subseteq (\tilde{H}, \wedge)$ & $(\tilde{L}, \wedge)$ is a N_sHSeos in $\mathfrak{M}\}$.

(ii) e closure (briefly, $N_sHSecl(\tilde{H}, \wedge)$ is defined by
$N_sHSecl(\tilde{H}, \wedge) = \bigcap\{(\tilde{L}, \wedge) : (\tilde{H}, \wedge) \subseteq (\tilde{L}, \wedge)$ & $(\tilde{H}, \wedge)$ is a N_sHSecs in $\mathfrak{M}\}$.

Proposition 4.3. The statements are correct, but the converse is not.

(i) Every N_sHSos (resp. N_sHScs) is a $N_sHS\delta\mathcal{S}os$ (resp. $N_sHS\delta\mathcal{S}cs$).
(ii) Every N_sHSos (resp. N_sHScs) is a $N_sHS\delta\mathcal{P}os$ (resp. $N_sHS\delta\mathcal{P}cs$).
(iii) Every $N_sHS\delta\mathcal{S}os$ (resp. $N_sHS\delta\mathcal{S}cs$) is a N_sHSeos (resp. N_sHSecs).
(iv) Every $N_sHS\delta\mathcal{P}os$ (resp. $N_sHS\delta\mathcal{P}cs$) is a N_sHSeos (resp. N_sHSecs).
(v) Every N_sHSeos (resp. N_sHSecs) is a N_sHSe^*os (resp. N_sHSe^*cs).

Proof. Consider,

(i) If $(\tilde{H}, \wedge)$ is a N_sHSos, then $(\tilde{H}, \wedge) = N_sHSint(\tilde{H}, \wedge)$. So, $(\tilde{H}, \wedge) = N_sHS$ $int(\tilde{H}, \wedge) \subseteq N_sHScl(N_sHS\delta int(\tilde{H}, \wedge))$. $\therefore$ $(\tilde{H}, \wedge)$ is a $N_sHS\delta\mathcal{S}os$.
(ii) If $(\tilde{H}, \wedge)$ is a N_sHSos, then $(\tilde{H}, \wedge) = N_sHSint(\tilde{H}, \wedge)$. So, $(\tilde{H}, \wedge) = N_sHS$ $int(\tilde{H}, \wedge) \subseteq N_sHSint(N_sHS\delta cl(\tilde{H}, \wedge))$. $\therefore$ $(\tilde{H}, \wedge)$ is a $N_sHS\delta\mathcal{P}os$.
(iii) If $(\tilde{H}, \wedge)$ is a $N_sHS\delta\mathcal{S}os$, then $(\tilde{H}, \wedge) \subseteq N_sHScl(N_sHS\delta int(\tilde{H}, \wedge)) \subseteq N_sHS$ $cl(N_sHS\delta int(\tilde{H}, \wedge)) \cup N_sHSint(N_sHS\delta cl(\tilde{H}, \wedge))$. $\therefore$ $(\tilde{H}, \wedge)$ is a N_sHSeos.
(iv) If $(\tilde{H}, \wedge)$ is a $N_sHS\delta\mathcal{P}os$, then $(\tilde{H}, \wedge) \subseteq N_sHSint(N_sHS\delta cl(\tilde{H}, \wedge)) \subseteq N_sHS$ $cl(N_sHS\delta int(\tilde{H}, \wedge)) \cup N_sHSint(N_sHS\delta cl(\tilde{H}, \wedge))$. $\therefore$ $(\tilde{H}, \wedge)$ is a N_sHSeos.
(v) If $(\tilde{H}, \wedge)$ is a N_sHSeos then $(\tilde{H}, \wedge) \subseteq N_sHScl\big(N_sHS\delta int(\tilde{H}, \wedge)\big) \cup N_sHS$ $int\big(N_sHS\delta cl(\tilde{H}, \wedge)\big)$. So $(\tilde{H}, \wedge) \subseteq N_sHScl\big(N_sHS\delta int(\tilde{H}, \wedge)\big) \cup N_sHSint$ $\big(N_sHS\delta cl(\tilde{H}, \wedge)\big) \subseteq N_sHScl\big(N_sHSint(N_sHS\delta cl(\tilde{H}, \wedge))\big)$. $\therefore$ $(\tilde{H}, \wedge)$ is a N_sHSe^*os.

This holds true for their closed sets as well. □

Remark 4.4. The diagram shows N_sHSeos's in N_sHSts.

Example 4.5. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2, \mathfrak{m}_3\}$ be a $NsHS$ initial universe and the attributes be Q_1, Q_2, Q_3. The attributes are given as:

$$Q_1 = \{a_1, a_2, a_3\},\ Q_2 = \{b_1, b_2\},\ Q_3 = \{c_1, c_2, c_3\}$$

Suppose that

$$E_1 = \{a_1, a_2\}, E_2 = \{b_1, b_2\}, E_3 = \{c_1, c_2\}$$
$$C_1 = \{a_1, a_2, a_3\}, C_2 = \{b_1, b_2\}, C_3 = \{c_1, c_2\}$$
$$D_1 = \{a_2, a_3\}, D_2 = \{b_1, b_2\}, D_3 = \{c_1\}$$

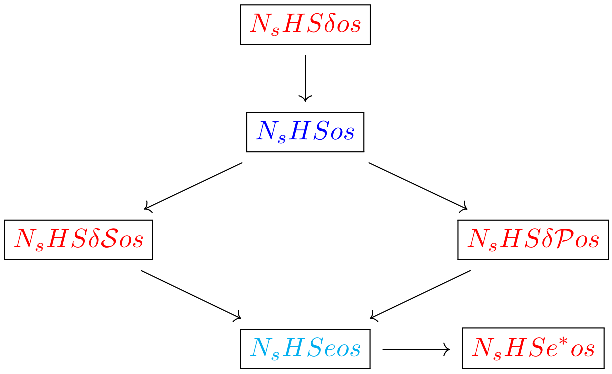

FIGURE 1. N_sHSeos's in N_sHSts

are subsets of Q_i for each $i = 1, 2, 3$. Then the N_sHSs $(\tilde{H}_1, \wedge_1)$, $(\tilde{H}_2, \wedge_2)$, $(\tilde{H}_3, \wedge_2)$, $(\tilde{H}_4, \wedge_2)$, $(\tilde{H}_5, \wedge_2)$ over the universe $\mathfrak{M}$ are as follows.

$$(\tilde{H}_1, \wedge_1) = \left\{ \begin{array}{l} \langle (a_1, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.8,0.1,0.9}, \frac{\mathfrak{m}_2}{0.3,0.2,0.3}, \frac{\mathfrak{m}_3}{0.2,0.2,0.6}\}\rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.7,0.4,0.8}, \frac{\mathfrak{m}_2}{0.7,0.3,0.8}, \frac{\mathfrak{m}_3}{0.5,0.5,0.8}\}\rangle \end{array} \right\}$$

$$(\tilde{H}_2, \wedge_2) = \left\{ \begin{array}{l} \langle (a_1, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.7,0.5,0.8}, \frac{\mathfrak{m}_2}{0.3,0.4,0.3}, \frac{\mathfrak{m}_3}{0.3,0.5,0.3}\}\rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.5,0.5,0.7}, \frac{\mathfrak{m}_2}{0.8,0.3,0.8}, \frac{\mathfrak{m}_3}{0.6,0.4,0.7}\}\rangle, \\ \langle (a_3, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.4,0.5,0.7}, \frac{\mathfrak{m}_2}{0.4,0.4,0.6}, \frac{\mathfrak{m}_3}{0.1,0.3,0.6}\}\rangle \end{array} \right\}$$

$$(\tilde{H}_3, \wedge_2) = \left\{ \begin{array}{l} \langle (a_1, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.8,0.5,0.8}, \frac{\mathfrak{m}_2}{0.3,0.4,0.3}, \frac{\mathfrak{m}_3}{0.3,0.5,0.3}\}\rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.7,0.5,0.7}, \frac{\mathfrak{m}_2}{0.8,0.3,0.8}, \frac{\mathfrak{m}_3}{0.6,0.5,0.7}\}\rangle, \\ \langle (a_3, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.4,0.5,0.7}, \frac{\mathfrak{m}_2}{0.4,0.4,0.6}, \frac{\mathfrak{m}_3}{0.1,0.3,0.6}\}\rangle \end{array} \right\}$$

$$(\tilde{H}_4, \wedge_2) = \left\{ \begin{array}{l} \langle (a_1, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.7,0.1,0.9}, \frac{\mathfrak{m}_2}{0.3,0.2,0.3}, \frac{\mathfrak{m}_3}{0.2,0.2,0.6}\}\rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.5,0.4,0.8}, \frac{\mathfrak{m}_2}{0.7,0.3,0.8}, \frac{\mathfrak{m}_3}{0.5,0.4,0.8}\}\rangle, \\ \langle (a_3, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.4,0.5,0.7}, \frac{\mathfrak{m}_2}{0.4,0.4,0.6}, \frac{\mathfrak{m}_3}{0.1,0.3,0.6}\}\rangle \end{array} \right\}$$

$$(\tilde{H}_5, \wedge_2) = \left\{ \begin{array}{l} \langle (a_1, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.8,0.1,0.9}, \frac{\mathfrak{m}_2}{0.3,0.2,0.3}, \frac{\mathfrak{m}_3}{0.2,0.2,0.6}\}\rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.7,0.4,0.8}, \frac{\mathfrak{m}_2}{0.7,0.3,0.8}, \frac{\mathfrak{m}_3}{0.5,0.5,0.8}\}\rangle, \\ \langle (a_3, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.4,0.5,0.7}, \frac{\mathfrak{m}_2}{0.4,0.4,0.6}, \frac{\mathfrak{m}_3}{0.1,0.3,0.6}\}\rangle \end{array} \right\}$$

$$(\tilde{H}_6, \wedge_3) = \left\{ \begin{array}{l} \langle (a_2, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.9,0.3,0.2}, \frac{\mathfrak{m}_2}{0.8,0.5,0.2}, \frac{\mathfrak{m}_3}{0.7,0.5,0.2}\}\rangle, \\ \langle (a_3, b_2, c_1), \{\frac{\mathfrak{m}_1}{0.8,0.5,0.3}, \frac{\mathfrak{m}_2}{0.7,0.6,0.1}, \frac{\mathfrak{m}_3}{0.9,0.6,0.2}\}\rangle \end{array} \right\}$$

$$(\tilde{H}_7, \wedge_1) = \left\{ \begin{array}{l} \langle (a_1, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.8,0.5,0.8}, \frac{\mathfrak{m}_2}{0.3,0.5,0.3}, \frac{\mathfrak{m}_3}{0.3,0.5,0.3}\}\rangle, \\ \langle (a_2, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.7,0.5,0.7}, \frac{\mathfrak{m}_2}{0.8,0.6,0.8}, \frac{\mathfrak{m}_3}{0.6,0.5,0.8}\}\rangle \end{array} \right\}$$

Then $\tau = \{0_{(\mathfrak{M},Q)}, 1_{(\mathfrak{M},Q)}, (\tilde{H}_1, \wedge_1), (\tilde{H}_2, \wedge_2), (\tilde{H}_3, \wedge_2), (\tilde{H}_4, \wedge_2), (\tilde{H}_5, \wedge_2)\}$ is a N_sHSts. Then,

(i) $(\tilde{H}_3, \wedge_2)^c$ is N_sHSeos but not $N_sHS\delta\mathcal{P}os$

(ii) $(\tilde{H}_1, \wedge_1)$ is N_sHSeos but not $N_sHS\delta\mathcal{S}os$
(iii) $(\tilde{H}_7, \wedge_1)$ is N_sHSe^*os but not N_sHSeos

Theorem 4.6. The statements are true.

(i) $N_sHS\delta\mathcal{P}cl(\tilde{H}, \wedge) \supseteq (\tilde{H}, \wedge) \cup N_sHScl\big(N_sHS\delta int(\tilde{H}, \wedge)\big)$.
(ii) $N_sHS\delta\mathcal{P}int(\tilde{H}, \wedge) \subseteq (\tilde{H}, \wedge) \cap N_sHSint\big(N_sHS\delta cl(\tilde{H}, \wedge)\big)$.
(iii) $N_sHS\delta\mathcal{S}cl(\tilde{H}, \wedge) \supseteq (\tilde{H}, \wedge) \cup N_sHSint\big(N_sHS\delta cl(\tilde{H}, \wedge)\big)$.
(iv) $N_sHS\delta\mathcal{S}int(\tilde{H}, \wedge) \subseteq (\tilde{H}, \wedge) \cap N_sHScl\big(N_sHS\delta int(\tilde{H}, \wedge)\big)$.

Proof. (i) Since $N_sHS\delta\mathcal{P}cl(\tilde{H}, \wedge)$ is $N_sHS\delta\mathcal{P}cs$, we have

$$\begin{aligned} N_sHScl(N_sHS\delta int(\tilde{H}, \wedge)) &\subseteq N_sHScl\big(N_sHS\delta int(N_sHS\delta\mathcal{P}cl(\tilde{H}, \wedge))\big) \\ &\subseteq N_sHS\delta\mathcal{P}cl(\tilde{H}, \wedge). \end{aligned}$$

Thus $(\tilde{H}, \wedge) \cup N_sHScl\big(N_sHS\delta int(\tilde{H}, \wedge)\big) \subseteq N_sHS\delta\mathcal{P}cl(\tilde{H}, \wedge)$. The other cases are similar. □

Theorem 4.7. $(\tilde{H}, \wedge)$ is a N_sHSeos iff $(\tilde{H}, \wedge) = N_sHS\delta\mathcal{P}int(\tilde{H}, \wedge) \cup N_sHS\delta\mathcal{S}int(\tilde{H}, \wedge)$.

Proof. Let $(\tilde{H}, \wedge)$ be a N_sHSeos. Then $(\tilde{H}, \wedge) \subseteq N_sHScl\big(N_sHS\delta int(\tilde{H}, \wedge)\big) \cup N_sHSint\big(N_sHS\delta cl(\tilde{H}, \wedge)\big)$. By Theorem 4.6, we have

$$\begin{aligned} &N_sHS\delta\mathcal{P}int(\tilde{H}, \wedge) \cup N_sHS\delta\mathcal{S}int(\tilde{H}, \wedge) \\ &\subseteq (\tilde{H}, \wedge) \cap \big(N_sHSint(N_sHS\delta cl(\tilde{H}, \wedge))\big) \cup \big((\tilde{H}, \wedge) \cap N_sHScl(N_sHS\delta int(\tilde{H}, \wedge))\big) \\ &= (\tilde{H}, \wedge) \cap \big(N_sHSint(N_sHS\delta cl(\tilde{H}, \wedge))\big) \cap N_sHScl\big(N_sHS\delta int(\tilde{H}, \wedge)\big) \\ &= (\tilde{H}, \wedge). \end{aligned}$$

Conversely, if $(\tilde{H}, \wedge) = N_sHS\delta\mathcal{P}int(\tilde{H}, \wedge) \cup N_sHS\delta\mathcal{S}int(\tilde{H}, \wedge)$ then, by Theorem 4.6

$$\begin{aligned} &(\tilde{H}, \wedge) = N_sHS\delta\mathcal{P}int(\tilde{H}, \wedge) \cup N_sHS\delta\mathcal{S}int(\tilde{H}, \wedge) \\ &\subseteq \big((\tilde{H}, \wedge) \cap N_sHSint(N_sHS\delta cl(\tilde{H}, \wedge))\big) \cup \big((\tilde{H}, \wedge) \cap N_sHScl(N_sHS\delta int(\tilde{H}, \wedge))\big) \\ &= (\tilde{H}, \wedge) \cap \big(N_sHSint(N_sHS\delta cl(\tilde{H}, \wedge)) \cup N_sHScl(N_sHS\delta int(\tilde{H}, \wedge))\big) \\ &\subseteq N_sHSint\big(N_sHS\delta cl(\tilde{H}, \wedge)\big) \cup N_sHScl\big(N_sHS\delta int(\tilde{H}, \wedge)\big) \end{aligned}$$

and hence $(\tilde{H}, \wedge)$ is a N_sHSeos. □

Theorem 4.8. The union (resp. intersection) of any family of $N_sHSeOS(\mathfrak{M})$ (resp. $N_sHSeCS(\mathfrak{M})$) is a $N_sHSeOS(\mathfrak{M})$ (resp. $N_sHSeCS(\mathfrak{M})$).

Proof. Let $\{(\tilde{H},\wedge)_a : a \in \tau\}$ be a family of N_sHSeos's. For each $a \in \tau$, $(\tilde{H},\wedge)_a \subseteq N_sHScl\big(N_sHS\delta int((\tilde{H},\wedge)_a)\big) \cup N_sHSint\big(N_sHS\delta cl((\tilde{H},\wedge)_a)\big)$.

$$\bigcup_{a\in\tau}(\tilde{H},\wedge)_a \subseteq \bigcup_{a\in\tau} N_sHScl\big(N_sHS\delta int((\tilde{H},\wedge)_a)\big) \cup N_sHSint\big(N_sHS\delta cl((\tilde{H},\wedge)_a)\big)$$
$$\subseteq N_sHScl\big(N_sHS\delta int(\cup(\tilde{H},\wedge)_a)\big) \cup N_sHSint\big(N_sHS\delta cl(\cup(\tilde{H},\wedge)_a)\big)$$

The other case is similar. □

Remark 4.9. The intersection of two N_sHSeos's need not be N_sHSeos.

Example 4.10. Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ be a $NsHS$ initial universe and the attributes be Q_1, Q_2, Q_3. The attributes are given as:

$$Q_1 = \{a_1, a_2\},\ Q_2 = \{b_1, b_2\},\ Q_3 = \{c_1, c_2\}$$

Suppose that

$$E_1 = \{a_1, a_2\}, E_2 = \{b_1, b_2\}, E_3 = \{c_1, c_2\}$$
$$C_1 = \{a_2\}, C_2 = \{b_1, b_2\}, C_3 = \{c_1, c_2\}$$

are subsets of Q_i for each $i = 1, 2, 3$. Then the N_sHSs $(\tilde{H}_1,\wedge_1)$, $(\tilde{H}_2,\wedge_2)$, $(\tilde{H}_3,\wedge_1)$ & $(\tilde{H}_4,\wedge_3)$ over the universe $\mathfrak{M}$ are as follows.

$$(\tilde{H}_1,\wedge_1) = \left\{\begin{matrix} \langle(a_1,b_1,c_1),\{\frac{\mathfrak{m}_1}{0.2,0.5,0.7},\frac{\mathfrak{m}_2}{0.1,0.5,0.5}\}\rangle, \\ \langle(a_2,b_2,c_2),\{\frac{\mathfrak{m}_1}{0.3,0.4,0.6},\frac{\mathfrak{m}_2}{0.2,0.5,0.6}\}\rangle \end{matrix}\right\}$$
$$(\tilde{H}_2,\wedge_2) = \left\{\begin{matrix} \langle(a_1,b_1,c_1),\{\frac{\mathfrak{m}_1}{0.3,0.5,0.7},\frac{\mathfrak{m}_2}{0.5,0.5,0.2}\}\rangle, \\ \langle(a_2,b_1,c_1),\{\frac{\mathfrak{m}_1}{0.4,0.4,0.5},\frac{\mathfrak{m}_2}{0.3,0.5,0.4}\}\rangle \end{matrix}\right\}$$
$$(\tilde{H}_3,\wedge_1) = \left\{\begin{matrix} \langle(a_1,b_1,c_1),\{\frac{\mathfrak{m}_1}{0.1,0.5,0.1},\frac{\mathfrak{m}_2}{0.2,0.5,0.1}\}\rangle, \\ \langle(a_2,b_2,c_2),\{\frac{\mathfrak{m}_1}{0.2,0.3,0.5},\frac{\mathfrak{m}_2}{0.1,0.5,0.6}\}\rangle \end{matrix}\right\}$$
$$(\tilde{H}_4,\wedge_3) = (\tilde{H}_2,\wedge_2) \cap (\tilde{H}_3,\wedge_1) = \left\{\begin{matrix} \langle(a_1,b_1,c_1),\{\frac{\mathfrak{m}_1}{0.1,0.5,0.7},\frac{\mathfrak{m}_2}{0.2,0.5,0.2}\}\rangle, \\ \langle(a_2,b_1,c_1),\{\frac{\mathfrak{m}_1}{0.4,0.4,0.5},\frac{\mathfrak{m}_2}{0.3,0.5,0.4}\}\rangle, \\ \langle(a_2,b_2,c_2),\{\frac{\mathfrak{m}_1}{0.2,0.3,0.5},\frac{\mathfrak{m}_2}{0.1,0.5,0.6}\}\rangle \end{matrix}\right\}$$

Then $\tau = \{0_{(\mathfrak{M},Q)}, 1_{(\mathfrak{M},Q)}, (\tilde{H}_1,\wedge_1)\}$ is a N_sHSts. Then $(\tilde{H}_2,\wedge_2)$ & $(\tilde{H}_3,\wedge_1)$ are N_sHSeos but $(\tilde{H}_2,\wedge_2) \cap (\tilde{H}_3,\wedge_1)$ is not N_sHSeos.

Proposition 4.11. If $(\tilde{H},\wedge)$ is a

(i) N_sHSeos and $N_sHS\delta int(\tilde{H},\wedge) = 0_{(\mathfrak{M},Q)}$, then $(\tilde{H},\wedge)$ is a $N_sHS\delta\mathcal{P}os$.
(ii) N_sHSeos and $N_sHS\delta cl(\tilde{H},\wedge) = 0_{(\mathfrak{M},Q)}$, then $(\tilde{H},\wedge)$ is a $N_sHS\delta\mathcal{S}os$.
(iii) N_sHSeos and $N_sHS\delta cs$, then $(\tilde{H},\wedge)$ is a $N_sHS\delta\mathcal{S}os$.
(iv) $N_sHS\delta\mathcal{S}os$ and $N_sHS\delta cs$, then $(\tilde{H},\wedge)$ is a N_sHSeos.

Proof. (i) Let $(\tilde{H},\wedge)$ be a N_sHSeos, that is

$$(\tilde{H},\wedge) \subseteq N_sHScl\big(N_sHS\delta int(\tilde{H},\wedge)\big) \cup N_sHSint\big(N_sHS\delta cl(\tilde{H},\wedge)\big)$$
$$= 0_{(\mathfrak{M},Q)} \cup N_sHSint\big(N_sHS\delta cl(\tilde{H},\wedge)\big)$$
$$= N_sHSint\big(N_sHS\delta cl(\tilde{H},\wedge)\big)$$

Hence $(\tilde{H},\wedge)$ is a $N_sHS\delta\mathcal{P}os$.

(ii) Let $(\tilde{H},\wedge)$ be a N_sHSeos, that is

$$\begin{aligned}(\tilde{H},\wedge) &\subseteq N_sHScl\big(N_sHS\delta int(\tilde{H},\wedge)\big)\cup N_sHSint\big(N_sHS\delta cl(\tilde{H},\wedge)\big)\\ &= N_sHScl\big(N_sHS\delta int(\tilde{H},\wedge)\big)\cup 0_{(\mathfrak{M},Q)}\\ &= N_sHScl\big(N_sHS\delta int(\tilde{H},\wedge)\big)\end{aligned}$$

Hence $(\tilde{H},\wedge)$ is a $N_sHS\delta\mathcal{S}os$.

(iii) Let $(\tilde{H},\wedge)$ be a N_sHSeos and $N_sHS\delta cs$, that is

$$\begin{aligned}(\tilde{H},\wedge) &\subseteq N_sHScl\big(N_sHS\delta int(\tilde{H},\wedge)\big)\cup N_sHSint\big(N_sHS\delta cl(\tilde{H},\wedge)\big)\\ &= N_sHScl\big(N_sHS\delta int(\tilde{H},\wedge)\big).\end{aligned}$$

Hence $(\tilde{H},\wedge)$ is a $N_sHS\delta\mathcal{S}os$.

(iv) Let $(\tilde{H},\wedge)$ be a $N_sHS\delta\mathcal{S}os$ and $N_sHS\delta cs$, that is

$$\begin{aligned}(\tilde{H},\wedge) &\subseteq N_sHScl\big(N_sHS\delta int(\tilde{H},\wedge)\big)\\ &\subseteq N_sHScl\big(N_sHS\delta int(\tilde{H},\wedge)\big)\cup N_sHSint\big(N_sHS\delta cl(\tilde{H},\wedge)\big).\end{aligned}$$

Hence $(\tilde{H},\wedge)$ is a N_sHSeos.

□

Theorem 4.12. $(\tilde{H},\wedge)$ is a N_sHSecs (resp. N_sHSeos) iff $(\tilde{H},\wedge) = N_sHSecl(\tilde{H},\wedge)$ (resp. $(\tilde{H},\wedge) = N_sHSeint(\tilde{H},\wedge)$).

Proof. Suppose $(\tilde{H},\wedge) = N_sHSecl(\tilde{H},\wedge) = \bigcap\{(\tilde{L},\wedge) : (\tilde{H},\wedge)\subseteq(\tilde{L},\wedge)\ \&\ (\tilde{L},\wedge)$ is a $N_sHSecs\}$. This means $(\tilde{H},\wedge)\in\bigcap\{(\tilde{L},\wedge) : (\tilde{H},\wedge)\subseteq(\tilde{L},\wedge)\ \&\ (\tilde{L},\wedge)$ is a $N_sHSecs\}$ and hence $(\tilde{H},\wedge)$ is N_sHSecs.

Conversely, suppose $(\tilde{H},\wedge)$ be a N_sHSecs in $\mathfrak{M}$. Then, we have $(\tilde{H},\wedge)\in\bigcap\{(\tilde{L},\wedge) : (\tilde{H},\wedge)\subseteq(\tilde{L},\wedge)\ \&\ (\tilde{L},\wedge)$ is a $N_sHSecs\}$. Hence, $(\tilde{H},\wedge)\subseteq(\tilde{L},\wedge)$ implies $(\tilde{H},\wedge) = \bigcap\{(\tilde{L},\wedge) : (\tilde{H},\wedge)\subseteq(\tilde{L},\wedge)\ \&\ (\tilde{L},\wedge)$ is a $N_sHSecs\} = N_sHSecl(\tilde{H},\wedge)$. Similarly $(\tilde{H},\wedge) = N_sHSeint(\tilde{H},\wedge)$.

□

Theorem 4.13. Let $(\tilde{H},\wedge)$ and $(\tilde{L},\wedge)$ in $\mathfrak{M}$, then the N_sHSecl sets have

(i) $N_sHSecl(0_{(\mathfrak{M},Q)}) = 0_{(\mathfrak{M},Q)}$, $N_sHSecl(1_{(\mathfrak{M},Q)}) = 1_{(\mathfrak{M},Q)}$.
(ii) $N_sHSecl(\tilde{H},\wedge)$ is a N_sHSecs in $\mathfrak{M}$.
(iii) $N_sHSecl(\tilde{H},\wedge)\subseteq N_sHSecl(\tilde{L},\wedge)$ if $(\tilde{H},\wedge)\subseteq(\tilde{L},\wedge)$.
(iv) $N_sHSecl\big(N_sHSecl(\tilde{H},\wedge)\big) = N_sHSecl(\tilde{H},\wedge)$.

Theorem 4.14. Let $(\tilde{H},\wedge)$ and $(\tilde{L},\wedge)$ in $\mathfrak{M}$, then the $N_sHSeint$ sets have

(i) $N_sHSeint(0_{(\mathfrak{M},Q)}) = 0_{(\mathfrak{M},Q)}$, $N_sHSeint(1_{(\mathfrak{M},Q)}) = 1_{(\mathfrak{M},Q)}$.
(ii) $N_sHSeint(\tilde{H},\wedge)$ is a N_sHSeos in $\mathfrak{M}$.
(iii) $N_sHSeint(\tilde{H},\wedge)\subseteq N_sHSeint(\tilde{L},\wedge)$ if $(\tilde{H},\wedge)\subseteq(\tilde{L},\wedge)$.

(iv) $N_sHSeint\big(N_sHSeint(\tilde{H},\wedge)\big) = N_sHSeint(\tilde{H},\wedge)$.

Proof. The proofs are directly from definitions of $N_sHSeint$ set. □

Proposition 4.15. Let $(\tilde{H},\wedge)$ and $(\tilde{L},\wedge)$ are in $\mathfrak{M}$, then

(i) $N_sHSecl(\tilde{H},\wedge)^c = [N_sHSeint(\tilde{H},\wedge)]^c$,
$N_sHSeint(\tilde{H},\wedge)^c = [N_sHSecl(\tilde{H},\wedge)]^c$.
(ii) $N_sHSecl\big((\tilde{H},\wedge)\cup(\tilde{L},\wedge)\big) \supseteq N_sHSecl(\tilde{H},\wedge)\cup N_sHSecl(\tilde{L},\wedge)$,
$N_sHSecl\big((\tilde{H},\wedge)\cap(\tilde{L},\wedge)\big) \subseteq N_sHSecl(\tilde{H},\wedge)\cap N_sHSecl(\tilde{L},\wedge)$.
(iii) $N_sHSeint\big((\tilde{H},\wedge)\cup(\tilde{L},\wedge)\big) \supseteq N_sHSeint(\tilde{H},\wedge)\cup N_sHSeint(\tilde{L},\wedge)$,
$N_sHSeint\big((\tilde{H},\wedge)\cap(\tilde{L},\wedge)\big) \subseteq N_sHSeint(\tilde{H},\wedge)\cap N_sHSeint(\tilde{L},\wedge)$.

Remark 4.16. As seen in the following example, the equality of (ii) in Proposition 4.15 does not have to be true.

Example 4.17. Consider the Example 4.10. Let

$$(\tilde{H}_5,\wedge_2) = \left\{\begin{matrix}\langle(a_1,b_1,c_1),\{\frac{\mathfrak{m}_1}{0.8,0.5,0.2},\frac{\mathfrak{m}_2}{0.6,0.6,0.1}\}\rangle,\\ \langle(a_2,b_1,c_1),\{\frac{\mathfrak{m}_1}{0.4,0.6,0.4},\frac{\mathfrak{m}_2}{0.7,0.4,0.4}\}\rangle\end{matrix}\right\}$$

$$(\tilde{H}_6,\wedge_3) = (\tilde{H}_3,\wedge_1)\cap(\tilde{H}_5,\wedge_2) = \left\{\begin{matrix}\langle(a_1,b_1,c_1),\{\frac{\mathfrak{m}_1}{0.1,0.5,0.2},\frac{\mathfrak{m}_2}{0.2,0.5,0.2}\}\rangle,\\ \langle(a_2,b_1,c_1),\{\frac{\mathfrak{m}_1}{0.4,0.6,0.4},\frac{\mathfrak{m}_2}{0.7,0.4,0.4}\}\rangle,\\ \langle(a_2,b_2,c_2),\{\frac{\mathfrak{m}_1}{0.2,0.3,0.5},\frac{\mathfrak{m}_2}{0.1,0.5,0.6}\}\rangle\end{matrix}\right\}$$

Then $N_sHSecl(\tilde{H}_3,\wedge_1) = 1_{(\mathfrak{M},Q)}$, $N_sHSecl(\tilde{H}_5,\wedge_2) = (\tilde{H}_5,\wedge_2)$ &
$N_sHSecl(\tilde{H}_3,\wedge_1)\cap N_sHSecl(\tilde{H}_5,\wedge_2) = (\tilde{H}_5,\wedge_2)$. Also, $N_sHSecl((\tilde{H}_3,\wedge_1)\cap(\tilde{H}_5,\wedge_2)) = N_sHSecl(\tilde{H}_6,\wedge_3) = (\tilde{H}_1,\wedge_1)^c$. Hence, $N_sHSecl((\tilde{H}_3,\wedge_1)\cap(\tilde{H}_5,\wedge_2)) \subsetneq N_sHSecl(\tilde{H}_3,\wedge_1)\cap N_sHSecl(\tilde{H}_5,\wedge_2)$.

Proposition 4.18. If $(\tilde{H},\wedge)$ is in $\mathfrak{M}$, then

(i) $N_sHSecl((\tilde{H},\wedge) \supseteq N_sHScl\big(N_sHS\delta int((\tilde{H},\wedge)\big)\cap N_sHSint\big(N_sHS\delta cl((\tilde{H},\wedge)\big)$.
(ii) $N_sHSeint((\tilde{H},\wedge) \subseteq N_sHScl\big(N_sHS\delta int((\tilde{H},\wedge)\big)\cup N_sHSint\big(N_sHS\delta cl((\tilde{H},\wedge)\big)$.

Proof. (i) $N_sHSecl((\tilde{H},\wedge)$ is a N_sHSecs and $((\tilde{H},\wedge) \subseteq N_sHSecl((\tilde{H},\wedge)$, then

$$\begin{aligned}N_sHSecl((\tilde{H},\wedge) &\supseteq N_sHScl\big(N_sHS\delta int(N_sHSecl((\tilde{H},\wedge))\big)\cap N_sHSint\\ &\qquad\big(N_sHS\delta cl(N_sHSecl((\tilde{H},\wedge))\big)\\ &\supseteq N_sHScl(N_sHS\delta int\big((\tilde{H},\wedge)\big)\cap N_sHSint\big(N_sHS\delta cl((\tilde{H},\wedge)\big).\end{aligned}$$

(ii) $N_sHSeint((\tilde{H},\wedge)$ is a N_sHSeos and $((\tilde{H},\wedge) \supseteq N_sHSeint((\tilde{H},\wedge)$, then

$$\begin{aligned}N_sHSeint((\tilde{H},\wedge) &\subseteq N_sHScl\big(N_sHS\delta int(N_sHSeint((\tilde{H},\wedge))\big)\cup N_sHSint\\ &\qquad\big(N_sHS\delta cl(N_sHSeint((\tilde{H},\wedge))\big)\\ &\subseteq N_sHScl\big(N_sHS\delta int((\tilde{H},\wedge)\big)\cup N_sHSint\big(N_sHS\delta cl((\tilde{H},\wedge)\big).\end{aligned}$$

□

Theorem 4.19. Let $((\tilde{H},\wedge)$ be in $\mathfrak{M}$, then

(i) $N_sHSecl((\tilde{H},\wedge) = N_sHS\delta\mathcal{P}cl((\tilde{H},\wedge) \cap N_sHS\delta\mathcal{S}cl(\tilde{H},\wedge)$.
(ii) $N_sHSeint(\tilde{H},\wedge) = N_sHS\delta\mathcal{P}int(\tilde{H},\wedge) \cap N_sHS\delta\mathcal{S}int(\tilde{H},\wedge)$.

Proof. (i) It is obvious that, $N_sHSecl(\tilde{H},\wedge) \subseteq N_sHS\delta\mathcal{P}cl(\tilde{H},\wedge) \cap N_sHS\delta\mathcal{S}\,cl(\tilde{H},\wedge)$. Conversely, from Definition 4.2, we have

$$\begin{aligned} N_sHSecl(\tilde{H},\wedge) &\supseteq N_sHScl\big(N_sHS\delta int(N_sHSecl(\tilde{H},\wedge))\big) \cap N_sHSint \\ &\qquad\qquad \big(N_sHS\delta cl(N_sHSecl(\tilde{H},\wedge))\big) \\ &\supseteq N_sHScl\big(N_sHS\delta int(\tilde{H},\wedge)\big) \cap N_sHSint\big(N_sHS\delta cl(\tilde{H},\wedge)\big). \end{aligned}$$

Since $N_sHSecl(\tilde{H},\wedge)$ is N_sHSecs, by Theorem 4.6, we have

$$\begin{aligned} &N_sHS\delta\mathcal{P}cl(\tilde{H},\wedge) \cap N_sHS\delta\mathcal{S}cl(\tilde{H},\wedge) \\ &\quad = (\tilde{H},\wedge) \cup N_sHScl\big(N_sHS\delta int(\tilde{H},\wedge)\big) \cap \big((\tilde{H},\wedge) \cup N_sHSint(N_sHS\delta cl(\tilde{H},\wedge))\big) \\ &\quad = (\tilde{H},\wedge) \cap \big(N_sHScl(N_sHS\delta int(\tilde{H},\wedge)) \cap N_sHSint(N_sHS\delta cl(\tilde{H},\wedge))\big) \\ &\quad = (\tilde{H},\wedge) \subseteq N_sHSecl(\tilde{H},\wedge). \end{aligned}$$

Therefore, $N_sHSecl(\tilde{H},\wedge) = N_sHS\delta\mathcal{P}cl(\tilde{H},\wedge) \cap N_sHS\delta\mathcal{S}cl(\tilde{H},\wedge)$. (ii) is similar from (i).

□

Theorem 4.20. Let $(\tilde{H},\wedge)$ be in $\mathfrak{M}$. Then

(i) $N_sHSecl(1_{(\mathfrak{M},Q)} - (\tilde{H},\wedge)) = 1_{(\mathfrak{M},Q)} - N_sHSeint(\tilde{H},\wedge)$.
(ii) $N_sHSeint(1_{(\mathfrak{M},Q)} - (\tilde{H},\wedge)) = 1_{(\mathfrak{M},Q)} - N_sHSecl(\tilde{H},\wedge)$.

5. Application in Covid-19 Diagnosis using Normalized Hamming Distance

In this section, normalized Hamming distance is applied in an example to diagnose Covid-19.

Example 5.1. Consider 2 patients visiting hospital with the following symptoms: Fever, Dry cough, Head ache, Body pain, Difficulty in breathing and Chest pain. The symptoms of Covid-19 patients can be categorized as
Severe symptoms = Difficulty in breathing, Chest pain
Most common symptoms = Fever, Dry cough
Less common symptoms = Headache, Body pain
Using the N_sHS model problem, the examination can be done whether the patients have the possibility of suffering from Covid-19 or not. Let $\mathfrak{M}$ be the universal set $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\} = \{\text{Covid-19, No Covid-19}\}$. The attributes are given as:

$$Q_1 = \{a_1 = \text{Difficulty in breathing},\ a_2 = \text{Chest pain}\}$$
$$Q_2 = \{b_1 = \text{Fever},\ b_2 = \text{Dry cough}\}$$
$$Q_3 = \{c_1 = \text{Headache},\ c_2 = \text{Body pain}\}$$

We define the N_sHSs's which give the degree of association, indeterminacy and the degree of non-association between the Covid-19 patients and the Covid-19 symptoms and between the 2 patients visited and their symptoms.
The N_sHSs $(\tilde{H}, \wedge)$ describes the evaluation of the Covid-19 patients and their symptoms as per the hospital records.

$$(\tilde{H}, \wedge) = \left\{ \begin{array}{l} \langle(a_1, b_1, c_1), \{\frac{\mathfrak{m}_1}{1.0,0.5,0.4}, \frac{\mathfrak{m}_2}{0.2,0.4,0.6}\}\rangle, \\ \langle(a_1, b_1, c_2), \{\frac{\mathfrak{m}_1}{0.9,0.4,0.2}, \frac{\mathfrak{m}_2}{0.1,0.6,0.7}\}\rangle, \\ \langle(a_1, b_2, c_1), \{\frac{\mathfrak{m}_1}{0.9,0.4,0.3}, \frac{\mathfrak{m}_2}{0.2,0.3,0.6}\}\rangle, \\ \langle(a_1, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.8,0.5,0.4}, \frac{\mathfrak{m}_2}{0.2,0.2,0.8}\}\rangle, \\ \langle(a_2, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.9,0.6,0.5}, \frac{\mathfrak{m}_2}{0.1,0.4,0.6}\}\rangle, \\ \langle(a_2, b_2, c_1), \{\frac{\mathfrak{m}_1}{0.8,0.7,0.3}, \frac{\mathfrak{m}_2}{0.1,0.6,0.5}\}\rangle, \\ \langle(a_2, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.8,0.5,0.4}, \frac{\mathfrak{m}_2}{0.1,0.5,0.7}\}\rangle, \\ \langle(a_2, b_1, c_2), \{\frac{\mathfrak{m}_1}{0.9,0.3,0.4}, \frac{\mathfrak{m}_2}{0.1,0.4,0.8}\}\rangle \end{array} \right\}$$

The N_sHSs's $(\tilde{G}, \wedge)$ & $(\tilde{P}, \wedge)$ describe the evaluation of the 2 patients visited and their symptoms respectively.

$$(\tilde{G}, \wedge) = \left\{ \begin{array}{l} \langle(a_1, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.1,0.5,0.9}, \frac{\mathfrak{m}_2}{0.9,0.2,0.6}\}\rangle, \\ \langle(a_1, b_1, c_2), \{\frac{\mathfrak{m}_1}{0.1,0.6,0.7}, \frac{\mathfrak{m}_2}{0.9,0.4,0.3}\}\rangle, \\ \langle(a_1, b_2, c_1), \{\frac{\mathfrak{m}_1}{0.0,0.5,0.8}, \frac{\mathfrak{m}_2}{0.9,0.6,0.4}\}\rangle, \\ \langle(a_1, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.1,0.4,0.7}, \frac{\mathfrak{m}_2}{0.9,0.7,0.3}\}\rangle, \\ \langle(a_2, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.2,0.5,0.8}, \frac{\mathfrak{m}_2}{0.9,0.3,0.5}\}\rangle, \\ \langle(a_2, b_2, c_1), \{\frac{\mathfrak{m}_1}{0.1,0.7,0.4}, \frac{\mathfrak{m}_2}{0.8,0.4,0.2}\}\rangle, \\ \langle(a_2, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.1,0.4,0.8}, \frac{\mathfrak{m}_2}{0.9,0.5,0.3}\}\rangle, \\ \langle(a_2, b_1, c_2), \{\frac{\mathfrak{m}_1}{0.1,0.5,0.7}, \frac{\mathfrak{m}_2}{0.9,0.3,0.2}\}\rangle \end{array} \right\}$$

$$(\tilde{P}, \wedge) = \left\{ \begin{array}{l} \langle(a_1, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.8,0.2,0.4}, \frac{\mathfrak{m}_2}{0.3,0.5,0.7}\}\rangle, \\ \langle(a_1, b_1, c_2), \{\frac{\mathfrak{m}_1}{0.7,0.6,0.2}, \frac{\mathfrak{m}_2}{0.2,0.5,0.7}\}\rangle, \\ \langle(a_1, b_2, c_1), \{\frac{\mathfrak{m}_1}{0.8,0.3,0.5}, \frac{\mathfrak{m}_2}{0.4,0.6,0.5}\}\rangle, \\ \langle(a_1, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.6,0.2,0.3}, \frac{\mathfrak{m}_2}{0.4,0.5,0.6}\}\rangle, \\ \langle(a_2, b_1, c_1), \{\frac{\mathfrak{m}_1}{0.8,0.5,0.1}, \frac{\mathfrak{m}_2}{0.2,0.6,0.6}\}\rangle, \\ \langle(a_2, b_2, c_1), \{\frac{\mathfrak{m}_1}{0.8,0.4,0.4}, \frac{\mathfrak{m}_2}{0.3,0.1,0.5}\}\rangle, \\ \langle(a_2, b_2, c_2), \{\frac{\mathfrak{m}_1}{0.7,0.5,0.3}, \frac{\mathfrak{m}_2}{0.3,0.4,0.6}\}\rangle, \\ \langle(a_2, b_1, c_2), \{\frac{\mathfrak{m}_1}{0.7,0.3,0.4}, \frac{\mathfrak{m}_2}{0.2,0.5,0.5}\}\rangle \end{array} \right\}$$

Using normalized Hamming distance, we get

$$d_{NH}((\tilde{H}, \wedge), (\tilde{G}, \wedge)) = 0.4167$$
$$d_{NH}((\tilde{H}, \wedge), (\tilde{P}, \wedge)) = 0.1458.$$

As the distance between the Covid-19 patient and the 2nd patient is lesser than 1st patient, there is larger possibility for the 2nd patient suffering from Covid-19.

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