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STUDY OF NEUTROSOPHIC MAGNIFIED TRANSLATION ON INCLINE ALGEBRA

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Abstract. The concept of magnified translation is studied in Neutrosophic set which is a segregation of mf (membership function) is known as t(truth)mf, i(indeterminate)mf, and f(falsity)mf which has been applied in incline algebra sub - structures and raises with a name called neutrosophic incline substructures are interlinked with the new frame of (nmt) neutrosophic magnified translation. Further, some more engrossing outputs such as homomorphic image, preimage and cartesian product on this neutrosophic magnified translation of incline substructure are investigated.

Key words: Incline algebra, subincline, ideal, neutrosophic set, translation and magnification.

1. INTRODUCTION

Fuzzy set(fs) by Zadeh [15] defines the concept of uncertainty in many real applications and this notion plays a vital role in the recent research. Later this concept is extended to interval valued fuzzy set(ivfs) with the membership function in terms of a collection of closed sub-interval of [0,1]. Atanassov [3] enlarged the fuzzy set by adding non membership for every element is known as intuitionistic fuzzy set(ifs). Further, Smarandache [12] explored a notation in which an middle term is added to the intuitionistic fuzzy set and it is named as neutrosophic set(ns). Many research is moving a long way with these types of sets and from those work this paper is motivated to work on this topic.

The structure of incline algebra is introduced by Cao, Kim and Roush [7, 8]. It is a generalization of boolean algebras, which is associative, commutative under addition & multiplication is distributive over addition with $\mathfrak{x}_1 + \mathfrak{x}_1 = \mathfrak{x}_1, \mathfrak{x}_1 + \mathfrak{x}_1\mathfrak{y}_1 = \mathfrak{x}_1, \mathfrak{y}_1 + \mathfrak{x}_1\mathfrak{y}_1 = \mathfrak{y}_1 \ \forall \mathfrak{x}_1, \mathfrak{y}_1$. Also covered with the poset & semiring structure . This incline algebra deals with different fields such as the graph theory, decision making,

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matrices etc., and the incline algebra was merged with fuzzy and its generalization by Jun [6].

Many algebraic structures are fused with fuzzy set started by Rosenfeld [11]. Further many algebras like BF−, BCK−, BCI− and B− algebras are successfully correlated with various kinds of fuzzy sets.

The idea of fuzzy translation(ft) and fuzzy multiplication(fm) was first started out by Lee et al. [10] in BCK / BCI algebras where various relations are discussed. In BF / BG - algebra the concept of fuzzy translation and multiplication was investigated by Chandramouleeswaran et al [4]. This concept is also discussed in cubic BCK / BCI subalgebras by Dutta et al. [5] and Khalid et al. [9] studied about translation(t) and multiplication(m) on neutrosophic cubic set.

With all these inspiration, this paper is merging the concept of incline algebra with neutrosophic set. Further the concept of translation, multliplication is applied in neutrosopic set also the notion called neutrosophic magnified translation of incline algebric sub - structures is investigated. Moreover, it deals about the cartesian product, homomorphic image, inverse image and some related results are disputed.

2. PRELIMINARIES

Fundamental defintions of incline algebra and fuzzy sets are discussed here in this section.

Definition 2.1. [15] *A function $\Phi : \mathfrak{U}$ to closed interval of 0,1 which is named as a fs in the univere $\mathfrak{U}$ where $\Phi(\mathfrak{a}_\circ)$ is the membership value of $\mathfrak{a}_\circ$ $\forall$ $\mathfrak{a}_\circ \in \mathfrak{U}$.*

Definition 2.2. [3] *An IFS in $\mathfrak{U}$ is represented as $\mathcal{B} = \{\mathfrak{a}_\circ, \Phi_{\mathcal{B}}(\mathfrak{a}_\circ), \omega_{\mathcal{B}}(\mathfrak{a}_\circ)/\mathfrak{a}_\circ \in \mathfrak{U}\}$ where $\Phi_{\mathcal{B}} : \mathfrak{U} \to [0,1]$ is a mf and $\Omega_{\mathcal{B}} : \mathfrak{U} \to [0,1]$ as nmf holds $0 \leq \Phi_{\mathcal{B}}(\mathfrak{a}_\circ) + \Omega_{\mathcal{B}}(\mathfrak{a}_\circ) \leq 1,\ \forall\ \mathfrak{a}_\circ \in \mathfrak{U}$.*

Definition 2.3. [12] *An neutrosophic set $\mathcal{W}$ is equal to $\{\mathfrak{a}_\circ, \Phi_{\mathcal{W}}(\mathfrak{a}_\circ), \Psi_{\mathcal{W}}(\mathfrak{a}_\circ), \omega_{\mathcal{W}}(\mathfrak{a}_\circ)$ $/\mathfrak{a}_\circ \in \mathfrak{U}\}$ on the universe $\mathfrak{U}$ with $\Phi_{\mathcal{W}}$ is tmf, $\Psi_{\mathcal{W}}$ is imf and $\Omega_{\mathcal{W}}$ is fmf where $\Phi_{\mathcal{R}}, \Psi_{\mathcal{R}}, \Omega_{\mathcal{R}} : \mathfrak{U} \to [0,1]$.*

Definition 2.4. [2, 14] *A non - empty set $(\mathfrak{I}, +, *)$ is an incline algebra if $\forall\ \mathfrak{a}_\circ, \mathfrak{b}_\circ \in \Upsilon$,*
(i) + in commutativity and + in associativity
(ii) Associative under $*$ and distributive (both left and right) under +
(iii) $\mathfrak{a}_\circ + \mathfrak{a}_\circ = \mathfrak{a}_\circ$ (idempotent)
(iv) $\mathfrak{a}_\circ + (\mathfrak{a}_\circ * \mathfrak{b}_\circ) = \mathfrak{a}_\circ$
(v) $\mathfrak{b}_\circ + (\mathfrak{a}_\circ * \mathfrak{b}_\circ) = \mathfrak{a}_\circ$.

Note:
In an incline the partial order is defined as $\mathfrak{a}_\circ \leq \mathfrak{b}_\circ \Leftrightarrow \mathfrak{a}_\circ + \mathfrak{b}_\circ = \mathfrak{b}_\circ$. Representing $\sqcap$ as min, $\sqcup$ as max respectively.

Definition 2.5. [2, 14] $\mathfrak{W} \subset \mathfrak{H}$ *of* $\mathfrak{I}$ *is an subincline if it is closed under* $+$ *and* $*$.

Definition 2.6. [2, 14] *A subincline* $\mathfrak{H}$ *of* $\mathfrak{I}$ *is an ideal if* $\mathfrak{a}_\circ \in \mathfrak{H}, \mathfrak{b}_\circ \in \mathfrak{I}$ *and* $\mathfrak{b}_\circ \leq \mathfrak{a}_\circ$ *then* $\mathfrak{b}_\circ \in \mathfrak{H}$.

Definition 2.7. [6] $\mathcal{A} = \{\mathfrak{a}_\circ, \Phi_{\mathcal{A}}(\mathfrak{a}_\circ)\}$ *is said to be a fuzzy subincline of incline algebra* $\mathfrak{I}$ *if* $\sqcap[\Phi_{\mathcal{A}}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), \Phi_{\mathcal{A}}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] \geq \sqcap[\Phi_{\mathcal{A}}(\mathfrak{a}_\circ), \Phi_{\mathcal{A}}(\mathfrak{b}_\circ)] \ \forall \ \mathfrak{a}_\circ, \mathfrak{b}_\circ \in \mathfrak{I}$.

Definition 2.8. [6] *An FS [fuzzy set]* $\mathcal{A}$ *is said to be order reversing if* $\Phi_{\mathcal{A}}(\mathfrak{a}_\circ) \geq \Phi_{\mathcal{A}}(\mathfrak{b}_\circ)$ *whenever* $\mathfrak{a}_\circ \leq \mathfrak{b}_\circ$.

Definition 2.9. [6] *The above two Definiitons 2.7 & 2.8 results out as fuzzy ideal of* $\mathfrak{I}$.

Definition 2.10. [4, 5] *Considering* Φ *as an fuzzy subset of* $\mathfrak{U}$, $\zeta \in [0, T]$ *where* $T = 1 - sup\{\Phi(\mathfrak{a}_\circ) : \mathfrak{a}_\circ \in \mathfrak{U}\}$. *A mapping* $\Phi^T_\zeta : \mathfrak{U} \to [0, 1]$ *is named as ft [fuzzy translation] of* Φ *if* $\Phi^T_\zeta(\mathfrak{a}_\circ) = \Phi(\mathfrak{a}_\circ) + \zeta \ \forall \mathfrak{a}_\circ \in \mathfrak{U}$.

Definition 2.11. [4, 5] *Considering* Φ *as a fuzzy subset of* $\mathfrak{U}$ *&* $\zeta \in [0, T]$. $\Phi^M_\zeta : \mathfrak{U} \to [0, 1]$ *mapping represents fm of* Φ *if it holds* $\Phi^M_\zeta(\mathfrak{a}_\circ) = \zeta\Phi(\mathfrak{a}_\circ) \ \forall \ \mathfrak{a}_\circ \in \mathfrak{U}$.

3. Neutrosophic Magnified Tranlsation on Incline Algebra

This section defines the definition of nmt with example as well its properties are discussed.

Definition 3.1. *Let* $\mathfrak{I}$ *be an incline algebra. The structure of the form* $\mathcal{W} = \{\mathfrak{a}_\circ, \Phi_{\mathcal{W}}(\mathfrak{a}_\circ), \Psi_{\mathcal{W}}(\mathfrak{a}_\circ), \Omega_{\mathcal{W}}(\mathfrak{a}_\circ)/\mathfrak{a}_\circ \in \mathfrak{I}\}$ *is said to be neutrosophic subincline of* $\mathfrak{I}$ *if,*
(i) $\sqcap[\Phi_{\mathcal{W}}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), \Phi_{\mathcal{W}}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] \geq \sqcap[\Phi_{\mathcal{W}}(\mathfrak{a}_\circ), \Phi_{\mathcal{W}}(\mathfrak{b}_\circ)]$,
(ii) $\sqcap[\Psi_{\mathcal{W}}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), \Psi_{\mathcal{W}}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] \geq \sqcap[\Psi_{\mathcal{W}}(\mathfrak{a}_\circ), \Psi_{\mathcal{W}}(\mathfrak{b}_\circ)]$,
(iii) $\sqcap[\Omega_{\mathcal{W}}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), \Phi_{\mathcal{W}}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] \leq \sqcap[\Omega_{\mathcal{W}}(\mathfrak{a}_\circ), \Omega_{\mathcal{W}}(\mathfrak{b}_\circ)]$ *for all* $\mathfrak{a}_\circ, \mathfrak{b}_\circ \in \mathfrak{I}$.

Definition 3.2. $\mathcal{W}$ *is known as neutrosophic ideal if it is an neutrosophic subincline and in addition if it satisfies the below axioms, whenever* $\mathfrak{a}_\circ \leq \mathfrak{b}_\circ$
(i) $\Phi_{\mathcal{W}}(\mathfrak{a}_\circ) \geq \Phi_{\mathcal{W}}(\mathfrak{b}_\circ)$
(ii) $\Psi_{\mathcal{W}}(\mathfrak{a}_\circ) \geq \Psi_{\mathcal{W}}(\mathfrak{b}_\circ)$
(iii) $\Omega_{\mathcal{W}}(\mathfrak{a}_\circ) \leq \Omega_{\mathcal{W}}(\mathfrak{b}_\circ)$.

Definition 3.3. *An neutrosophic set* $\mathcal{W}$ *of* $\mathfrak{I}$ *and* $\zeta \in [0, 1]$, *then the object* $\mathcal{W}^M_\zeta = \{(\Phi_{\mathcal{W}})^M_\zeta, (\Psi_{\mathcal{W}})^M_\zeta, (\omega_{\mathcal{W}})^M_\zeta\}$ *is known as nm [neutrosophic multiplication] of* $\mathfrak{I}$, *where* $(\Phi_{\mathcal{W}})^M_\zeta(\mathfrak{a}_\circ) = \zeta\Phi_{\mathcal{W}}(\mathfrak{a}_\circ), (\Psi_{\mathcal{W}})^M_\zeta(\mathfrak{a}_\circ) = \zeta\Psi_{\mathcal{W}}(\mathfrak{a}_\circ), (\omega_{\mathcal{W}})^M_\zeta(\mathfrak{a}_\circ) = \zeta\omega_{\mathcal{W}}(\mathfrak{a}_\circ) \ \forall \ \mathfrak{a}_\circ \in \mathfrak{I}$.

Definition 3.4. *Let* $\mathcal{W}$ *an neutrosophic set of* $\mathfrak{I}$ *and* $p', q', r' \in [0, \mathfrak{N}]$, *an structure of the form* $\mathcal{W}^T_{p',q',r'} = \{(\Phi_{\mathcal{W}})^T_{p'}, (\Psi_{\mathcal{W}})^T_{q'}, (\omega_{\mathcal{W}})^T_{r'}\}$ *is named as nt [neutrosophic translation] of* $\mathfrak{I}$, *where* $(\Phi_{\mathcal{W}})^T_{p'}(\mathfrak{a}_\circ) = \Phi_{\mathcal{W}}(\mathfrak{a}_\circ) + p', (\Psi_{\mathcal{W}})^T_{q'}(\mathfrak{a}_\circ) = \Psi_{\mathcal{W}}(\mathfrak{a}_\circ) + q', (\omega_{\mathcal{W}})^{MT}_{r'}(\mathfrak{a}_\circ) = \omega_{\mathcal{W}}(\mathfrak{a}_\circ) - r' \ \forall \ \mathfrak{a}_\circ \in \mathfrak{I}$.

Definition 3.5. *Let* $\mathcal{W} = \{\mathfrak{a}_\circ, \Phi_{\mathcal{W}}(\mathfrak{a}_\circ), \Psi_{\mathcal{W}}(\mathfrak{a}_\circ), \omega_{\mathcal{W}}(\mathfrak{a}_\circ) : \mathfrak{a}_\circ \in \mathfrak{I}\}$ *be a neutrosophic set of* $\mathfrak{I}$ *and* $p^{'}, q^{'}, r^{'} \in [0, \mathfrak{N}]$, $\zeta \in [0,1]$. *An structure of the form* $\mathcal{W}^{MT}_{\zeta,p^{'},q^{'},r^{'}} = \{(\Phi_{\mathcal{W}})^{MT}_{\zeta,p^{'}}, (\Psi_{\mathcal{W}})^{MT}_{\zeta,q^{'}}, (\omega_{\mathcal{W}})^{MT}_{\zeta,r^{'}}\}$ *is said to be a nmt of* $\mathfrak{I}$, *where* $(\Phi_{\mathcal{W}})^{MT}_{\zeta,p^{'}}(\mathfrak{a}_\circ) = \zeta\Phi_{\mathcal{W}}(\mathfrak{a}_\circ) + p^{'}$, $(\Psi_{\mathcal{W}})^{MT}_{\zeta,q^{'}}(\mathfrak{a}_\circ) = \zeta\Psi_{\mathcal{W}}(\mathfrak{a}_\circ) + q^{'}$, $(\omega_{\mathcal{W}})^{MT}_{\zeta,r^{'}}(\mathfrak{a}_\circ) = \zeta\omega_{\mathcal{W}}(\mathfrak{a}_\circ) - r^{'}$ $\forall\ \mathfrak{a}_\circ \in \mathfrak{I}$.

Example 3.6. $\mathfrak{I} = \{\epsilon^{'}, \mathfrak{x}^{'}, \mathfrak{y}^{'}, \mathfrak{z}^{'}\}$ *is an incline with binary operation* $+$ *and* $*$ *defined on* $\mathfrak{I}$ *in the below cayley table*

$+$	$\epsilon^{'}$	$\mathfrak{x}^{'}$	$\mathfrak{y}^{'}$	$\mathfrak{z}^{'}$
$\epsilon^{'}$	$\epsilon^{'}$	$\mathfrak{x}^{'}$	$\mathfrak{y}^{'}$	$\mathfrak{z}^{'}$
$\mathfrak{x}^{'}$	$\mathfrak{x}^{'}$	$\mathfrak{x}^{'}$	$\mathfrak{x}^{'}$	$\mathfrak{x}^{'}$
$\mathfrak{y}^{'}$	$\mathfrak{y}^{'}$	$\mathfrak{x}^{'}$	$\mathfrak{y}^{'}$	$\mathfrak{x}^{'}$
$\mathfrak{z}^{'}$	$\mathfrak{z}^{'}$	$\mathfrak{x}^{'}$	$\mathfrak{x}^{'}$	$\mathfrak{z}^{'}$

$*$	$\epsilon^{'}$	$\mathfrak{x}^{'}$	$\mathfrak{y}^{'}$	$\mathfrak{z}^{'}$
$\epsilon^{'}$	$\epsilon^{'}$	$\epsilon^{'}$	$\epsilon^{'}$	$\epsilon^{'}$
$\mathfrak{x}^{'}$	$\epsilon^{'}$	$\mathfrak{x}^{'}$	$\mathfrak{y}^{'}$	$\mathfrak{z}^{'}$
$\mathfrak{y}^{'}$	$\epsilon^{'}$	$\mathfrak{y}^{'}$	$\mathfrak{y}^{'}$	$\epsilon^{'}$
$\mathfrak{z}^{'}$	$\epsilon^{'}$	$\mathfrak{z}^{'}$	$\epsilon^{'}$	$\mathfrak{z}^{'}$

Table 1

Now defining a neutrosophic set $\mathcal{W}$ *on* $\mathfrak{I}$ *with it's membership functions as follows*

$$\Phi_{\mathcal{W}}(\mathfrak{a}_\circ) = \begin{cases} 0.4: & \mathfrak{a}_\circ = \epsilon^{'}, \mathfrak{z}^{'} \\ 0.3: & \mathfrak{a}_\circ = \mathfrak{x}^{'}, \mathfrak{y}^{'} \end{cases} \quad \Psi_{\mathcal{W}}(u_\circ) = \begin{cases} 0.6: & \mathfrak{a}_\circ = \epsilon^{'}, \mathfrak{z}^{'} \\ 0.2: & \mathfrak{a}_\circ = \mathfrak{x}^{'}, \mathfrak{y}^{'} \end{cases} \quad \Omega_{\mathcal{W}}(\mathfrak{a}_\circ) = \begin{cases} 0.1: & \mathfrak{a}_\circ = \epsilon^{'}, \mathfrak{z}^{'} \\ 0.5: & \mathfrak{a}_\circ = \mathfrak{x}^{'}, \mathfrak{y}^{'} \end{cases}$$

Thus, $\mathcal{W}$ *is a neutrosophic subincline of* $\mathfrak{I}$.
Consider a subset $\mathfrak{W} = \{\mathfrak{y}^{'}, \mathfrak{z}^{'}\}$ *of* $\mathfrak{I}$ *which is defined by the following membership functions,*

$$\Phi_{\mathfrak{W}}(\mathfrak{a}_\circ) = \begin{cases} 0.8: & \mathfrak{a}_\circ = \mathfrak{y}^{'} \\ 0.7: & \mathfrak{a}_\circ = \mathfrak{z}^{'} \end{cases} \quad \Psi_{\mathfrak{W}}(\mathfrak{a}_\circ) = \begin{cases} 0.6: & \mathfrak{a}_\circ = \mathfrak{y}^{'} \\ 0.5: & \mathfrak{a}_\circ = \mathfrak{z}^{'} \end{cases} \quad \Omega_{\mathfrak{W}}(\mathfrak{a}_\circ) = \begin{cases} 0.3: & \mathfrak{a}_\circ = \mathfrak{y}^{'} \\ 0.45: & \mathfrak{a}_\circ = \mathfrak{z}^{'} \end{cases}$$

Thus, $\mathfrak{W}$ *is an neutrosophic ideal of* $\mathfrak{I}$.
Choose, $\zeta = 0.3$, $p^{'} = 0.04$, $q^{'} = 0.05$, $r^{'} = 0.06$ *then the mapping* $\mathcal{W}^{MT}_{(0.3)(0.04,0.05,0.06)} : \mathfrak{I} \to [0,1]$ *is given by*

$$(\Phi_{\mathcal{W}})^{MT}_{0.3,0.04}(\mathfrak{a}_\circ) = \begin{cases} 0.08: & \mathfrak{a}_\circ = \epsilon^{'}, \mathfrak{z}^{'} \\ 0.15: & \mathfrak{a}_\circ = \mathfrak{x}^{'}, \mathfrak{y}^{'} \end{cases}$$

$$(\Psi_{\mathcal{W}})^{MT}_{0.3,0.05}(\mathfrak{a}_\circ) = \begin{cases} 0.09: & \mathfrak{a}_\circ = \epsilon^{'}, \mathfrak{z}^{'} \\ 0.20: & \mathfrak{a}_\circ = \mathfrak{x}^{'}, \mathfrak{y}^{'} \end{cases}$$

$$(\Omega_{\mathcal{W}})^{MT}_{0.3,0.06}(\mathfrak{a}_\circ) = \begin{cases} 0.03: & \mathfrak{a}_\circ = \epsilon^{'}, \mathfrak{z}^{'} \\ 0.02: & \mathfrak{a}_\circ = \mathfrak{x}^{'}, \mathfrak{y}^{'} \end{cases}$$

If $\zeta = 1$, $p^{'} = 0 = q^{'} = r^{'}$, *then* $\mathcal{W}^{MT}_{(1)(0,0,0)}$ *is a neutrosophic set.*
If $\zeta = 0.3$, $p^{'} = q^{'} = r^{'} = 0$, *then* $\mathcal{W}^{MT}_{(0.3)(0,0,0)}$ *is a nm.*
If $\zeta = 1$, $p^{'} = 0.04$, $q^{'} = 0.05$, $r^{'} = 0.06$, $\mathcal{W}^{MT}_{(1)(0.04,0.05,0.06)}$ *is said to be nt.*
Hence $\mathcal{W}^{MT}_{(0.3)(0.04,0.05,0.06)}$ *is a nmt (neutrosophic maginified translation).*

Theorem 3.7. *Let $\mathcal{W} = \{\mathfrak{a}_\circ, \Phi_{\mathcal{W}}(\mathfrak{a}_\circ), \Psi_{\mathcal{W}}(\mathfrak{a}_\circ), \Omega_{\mathcal{W}}(\mathfrak{a}_\circ) : \mathfrak{a}_\circ \in \mathfrak{I}\}$ be a neutrosophic subset of $\mathfrak{I}$ such that $p^{'}, q^{'}, r^{'} \in [0, \mathfrak{N}]$, $\zeta \in [0,1]$ and a mapping $\mathcal{W}^{MT}_{\zeta,p^{'},q^{'},r^{'}} : \mathfrak{I} \to [0,1]$ be a nmt of $\mathfrak{I}$. If $\mathcal{W}$ of $\mathfrak{I}$ is a neutrosophic subincline(ideal), then $\mathcal{W}^{MT}_{\zeta,p^{'},q^{'},r^{'}}$ of $\mathfrak{I}$ is referred as neutrosophic subincline(ideal).*

Proof. $\mathcal{W}$ is a neutrosophic set of $\mathfrak{I}, p^{'}, q^{'}, r^{'} \in [0, \mathfrak{N}], \zeta \in [0,1]$ and $\mathcal{W}^{MT}_{\zeta,p^{'},q^{'},r^{'}} : \mathfrak{I} \to [0,1]$ be a nmt [neutrosophic maginified translation] of $\mathfrak{I}$. If $\mathcal{W}$ is a neutrosophic subincline(ideal) of $\mathfrak{I}$, then

$$\begin{aligned}
\sqcap[(\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), (\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] &= \sqcap[\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ + \mathfrak{b}_\circ) + p^{'}, \zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ * \mathfrak{b}_\circ) + p^{'}] \\
&\geq \sqcap[\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ) + p^{'}, \zeta(\Phi_{\mathcal{W}})(\mathfrak{b}_\circ) + p^{'}] \\
&= \sqcap[(\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ), (\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{b}_\circ)] \\
\sqcap[(\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), (\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] &\geq \sqcap[(\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ), (\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{b}_\circ)]
\end{aligned}$$

Whenever $u_\circ \leq v_\circ$

$$\begin{aligned}
(\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ) &= \zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ) + p^{'} \\
&\geq \zeta(\Phi_{\mathcal{W}})(\mathfrak{b}_\circ) + p^{'} \\
&= (\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{b}_\circ) \\
(\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{b}_\circ) &\geq (\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{b}_\circ)
\end{aligned}$$

Similarly, $\sqcap[(\Psi_{\mathcal{W}})^{MT}_{\zeta,q'}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), (\Phi_{\mathcal{W}})^{MT}_{\zeta,q'}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] \geq \sqcap[(\Psi_{\mathcal{W}})^{MT}_{\zeta,q'}(\mathfrak{a}_\circ), (\Psi_{\mathcal{W}})^{MT}_{\zeta,q'}(\mathfrak{b}_\circ)]$
and
$(\Psi_{\mathcal{W}})^{MT}_{\zeta,q'}(\mathfrak{a}_\circ) \geq (\Psi_{\mathcal{W}})^{MT}_{\zeta,q'}(\mathfrak{b}_\circ)$

$$\begin{aligned}
\sqcup[(\Omega_{\mathcal{W}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), (\Omega_{\mathcal{W}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] &= \sqcup[\zeta(\Omega_{\mathcal{W}})(\mathfrak{a}_\circ + \mathfrak{b}_\circ) - r^{'}, \zeta(\Omega_{\mathcal{W}})(\mathfrak{a}_\circ * \mathfrak{b}_\circ) - r^{'}] \\
&\leq \sqcup[\zeta(\Omega_{\mathcal{W}})(\mathfrak{a}_\circ) - r^{'}, \zeta(\Omega_{\mathcal{W}})(\mathfrak{b}_\circ) - r^{'}] \\
&= \sqcup[(\Omega_{\mathcal{W}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ), (\Omega_{\mathcal{W}})^{MT}_{\zeta,r'}(\mathfrak{b}_\circ)] \\
\sqcup[(\Omega_{\mathcal{W}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), (\Omega_{\mathcal{W}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] &\leq \sqcup[(\Omega_{\mathcal{W}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ), (\Omega_{\mathcal{W}})^{MT}_{\zeta,r'}(\mathfrak{b}_\circ)] \\
(\Omega_{\mathcal{W}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ) &= \zeta(\Omega_{\mathcal{W}})(\mathfrak{a}_\circ) - r^{'} \\
&\leq \zeta(\Omega_{\mathcal{W}})(\mathfrak{b}_\circ) - r^{'} \\
&= (\Omega_{\mathcal{W}})^{MT}_{\zeta,r'}(\mathfrak{b}_\circ) \\
(\Omega_{\mathcal{W}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ) &\leq (\Omega_{\mathcal{W}})^{MT}_{\zeta,r'}(\mathfrak{b}_\circ)
\end{aligned}$$

Thus, $\mathcal{W}^{MT}_{\zeta,p^{'},q^{'},r^{'}}$ states that it's an neutrosophic subincline(ideal) of $\mathfrak{I}$. □

Theorem 3.8. *Consider $\mathcal{W}$ as an ns of $\mathfrak{I}, p^{'}, q^{'}, r^{'} \in [0, \mathfrak{N}]$, $\zeta \in [0,1]$ and a mapping $\mathcal{W}^{MT}_{\zeta,p^{'},q^{'},r^{'}} : \mathfrak{I} \to [0,1]$ be a nmt of $\mathfrak{I}$. If $\mathcal{W}^{MT}_{\zeta,p^{'},q^{'},r^{'}}$ of $\mathfrak{I}$ is a neutrosophic subincline[ideal], then $\mathcal{W}$ of $\mathfrak{I}$ is a neutrosophic subincline[ideal].*

Proof. An ns $\mathcal{W}$ of $\mathfrak{I}, p^{'}, q^{'}, r^{'} \in [0, \mathfrak{N}], \zeta \in [0,1]$ and $\mathcal{W}^{MT}_{\zeta,p^{'},q^{'},r^{'}} : \mathfrak{I} \to [0,1]$ be a nmt of $\mathfrak{I}$. If $\mathcal{W}^{MT}_{\zeta,p^{'},q^{'},r^{'}}$ is a neutrosophic subincline(ideal) of $\mathfrak{I}$, then

$$\begin{aligned}\sqcap[\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ + \mathfrak{b}_\circ) + p^{'}, \zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ * \mathfrak{b}_\circ) + p^{'}] &= \sqcap[(\Phi_{\mathcal{W}})^{MT}_{\zeta,p^{'}}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), (\Phi_{\mathcal{W}})^{MT}_{\zeta,p^{'}}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)]\\ &\geq \sqcap[(\Phi_{\mathcal{W}})^{MT}_{\zeta,p^{'}}(\mathfrak{a}_\circ), (\Phi_{\mathcal{W}})^{MT}_{\zeta,p^{'}}(\mathfrak{b}_\circ)]\\ &= \sqcap[\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ) + p^{'}, \zeta(\Phi_{\mathcal{W}})(\mathfrak{b}_\circ) + p^{'}]\end{aligned}$$

Implies, $\sqcap[\Phi_{\mathcal{W}}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), \Phi_{\mathcal{W}}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] \geq \sqcap[\Phi_{\mathcal{W}}(u_\circ), \Phi_{\mathcal{W}}(\mathfrak{b}_\circ)]$.
Whenever $\mathfrak{a}_\circ \leq \mathfrak{b}_\circ$

$$\begin{aligned}\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ) + p^{'} &= (\Phi_{\mathcal{W}})^{MT}_{\zeta,p^{'}}(\mathfrak{a}_\circ)\\ &\geq (\Phi_{\mathcal{W}})^{MT}_{\zeta,p^{'}}(\mathfrak{b}_\circ)\\ &= \zeta(\Phi_{\mathcal{W}})(\mathfrak{b}_\circ) + p^{'}\end{aligned}$$

Thus, $\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ) \geq \zeta(\Phi_{\mathcal{W}})(\mathfrak{b}_\circ)$. Likewise, for the other components. Hence $\mathcal{W}$ is a neutrosophic subincline(ideal) of $\mathfrak{I}$. ☐

Theorem 3.9. *The intersection of any two nmt $\mathcal{W}^{MT}_{\zeta,p^{'},q^{'},r^{'}}$ of $\mathfrak{I}$ is an neutrosophic subincline[ideal] of $\mathcal{W}$ of $\mathfrak{I}$ is a neutrosophic subincline[ideal].*

Proof. Considering $\mathcal{W}^{MT}_{\zeta,p^{'},q^{'},r^{'}}$ and $\mathcal{W}^{MT}_{\zeta_\circ,p^{'}_\circ,q^{'}_\circ,r^{'}_\circ}$ be two nmt of neutrosophic subincline(ideal) of $\mathfrak{I}$. Assume that $p^{'} \leq p^{'}_\circ, q^{'} \leq q^{'}_\circ, r^{'} \leq r^{'}_\circ, \zeta = \zeta_\circ$. Since $\mathcal{W}^{MT}_{\zeta,p^{'},q^{'},r^{'}}$ and $\mathcal{W}^{MT}_{\zeta_\circ,p^{'}_\circ,q^{'}_\circ,r^{'}_\circ}$ are two neutrosophic subincline of $\mathfrak{I}$.
Consider, $(\Phi^{MT}_{\mathcal{W}})_{(\zeta,p^{'})\cap(\zeta_\circ,p^{'}_\circ)}(\mathfrak{a}_\circ + \mathfrak{b}_\circ)$

$$\begin{aligned}((\Phi^{MT}_{\mathcal{W}})_{(\zeta,p^{'})\cap(\zeta_\circ,p^{'}_\circ)}(\mathfrak{a}_\circ + \mathfrak{b}_\circ)) &= \sqcap[(\Phi_{\mathcal{W}})^{MT}_{\zeta,p^{'}}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), (\Phi_{\mathcal{W}})^{MT}_{\zeta_\circ,p^{'}_\circ}(\mathfrak{a}_\circ + \mathfrak{b}_\circ)]\\ &= \sqcap[\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ + \mathfrak{b}_\circ) + p^{'}, \zeta_\circ(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ + \mathfrak{b}_\circ) + p^{'}_\circ]\\ &\geq \sqcap[\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ) + p^{'}, \zeta_\circ(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ) + p^{'}_\circ]\\ &= (\Phi^{MT}_{\mathcal{W}})_{(\zeta,p^{'})\cap(\zeta_\circ,p^{'}_\circ)}(\mathfrak{a}_\circ).\end{aligned}$$

Now $(\Phi^{MT}_{\mathcal{W}})_{(\zeta,p^{'})\cap(\zeta_\circ,p^{'}_\circ)}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)$

$$\begin{aligned}((\Phi^{MT}_{\mathcal{W}})_{(\zeta,p^{'})\cap(\zeta_\circ,p^{'}_\circ)}(\mathfrak{a} * \mathfrak{b})) &= \sqcap[(\Phi_{\mathcal{W}})^{MT}_{\zeta,p^{'}}(\mathfrak{a} * \mathfrak{b}), (\Phi_{\mathcal{W}})^{MT}_{\zeta_\circ,p^{'}_\circ}(\mathfrak{a} * \mathfrak{b})]\\ &= \sqcap[\zeta(\Phi_{\mathcal{W}})(\mathfrak{a} * \mathfrak{b}) + p^{'}, \zeta_\circ(\Phi_{\mathcal{W}})(\mathfrak{a} * \mathfrak{b}) + p^{'}_\circ]\\ &\geq \sqcap[\zeta(\Phi_{\mathcal{W}})(\mathfrak{b}) + p^{'}, \zeta_\circ(\Phi_{\mathcal{W}})(\mathfrak{b}) + p^{'}_\circ]\\ &= (\Phi^{MT}_{\mathcal{W}})_{(\zeta,p^{'})\cap(\zeta_\circ,p^{'}_\circ)}(\mathfrak{b}).\end{aligned}$$

Thus, $\sqcap[(\Phi^{MT}_{\mathcal{W}})_{(\zeta,p^{'})\cap(\zeta_\circ,p^{'}_\circ)}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), (\Phi^{MT}_{\mathcal{W}})_{(\zeta,p^{'})\cap(\zeta_\circ,p^{'}_\circ)}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)]$
$\geq \sqcap(\Phi^{MT}_{\mathcal{W}})_{(\zeta,p^{'})\cap(\zeta_\circ,p^{'}_\circ)}(\mathfrak{a}_\circ), (\Phi^{MT}_{\mathcal{W}})_{(\zeta,p^{'})\cap(\zeta_\circ,p^{'}_\circ)}(\mathfrak{b}_\circ)$.

And,

$$\begin{aligned}(\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cap(\zeta_\circ,p'_\circ)}(\mathfrak{a}_\circ) &= \sqcap[\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ)+p^{'},\zeta_\circ(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ)+p_\circ^{'}]\\ &\geq \sqcap[\zeta(\Phi_{\mathcal{W}})(\mathfrak{b}_\circ)+p^{'},\zeta_\circ(\Phi_{\mathcal{W}})(\mathfrak{b}_\circ)+p_\circ^{'}]\\ &= (\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cap(\zeta_\circ,p'_\circ)}(\mathfrak{b}_\circ)\end{aligned}$$

Analogously, for the other two membership function. Thus, $\mathcal{W}^{MT}_{\zeta,p',q',r'} \cap \mathcal{W}^{MT}_{\zeta_\circ,p'_\circ,q'_\circ,r'_\circ}$ is a neutrosophic subincline(ideal) of $\mathfrak{I}$. □

Theorem 3.10. *Union of any two neutrosophic magnified translation $\mathcal{W}^{MT}_{\zeta,p',q',r'}$ of $\mathcal{W}$ of $\mathfrak{I}$ is a neutrosophic subincline(ideal).*

Proof. Let $\mathcal{W}^{MT}_{\zeta,p',q',r'}$ and $\mathcal{W}^{MT}_{\zeta_\circ,p'_\circ,q'_\circ,r'_\circ}$ be two neutrosophic subincline(ideal) of $\mathfrak{I}$. Take $(\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cup(\zeta_\circ,p'_\circ)}(\mathfrak{a}_\circ+\mathfrak{b}_\circ)$

$$\begin{aligned}((\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cup(\zeta_\circ,p'_\circ)}(\mathfrak{a}_\circ+\mathfrak{b}_\circ)) &= \sqcup[(\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ+\mathfrak{b}_\circ),(\Phi_{\mathcal{W}})^{MT}_{\zeta_\circ,p'_\circ}(\mathfrak{a}+\mathfrak{b})]\\ &= \sqcup[\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ+\mathfrak{b}_\circ)+p^{'},\zeta_\circ(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ+\mathfrak{b}_\circ)+p_\circ^{'}]\\ &\geq \sqcup[\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ)+p^{'},\zeta_\circ(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ)+p_\circ^{'}]\\ &= (\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cup(\zeta_\circ,p'_\circ)}(\mathfrak{a}_\circ).\end{aligned}$$

Now $(\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cup(\zeta_\circ,p'_\circ)}(\mathfrak{a}_\circ*\mathfrak{b}_\circ)$

$$\begin{aligned}((\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cup(\zeta_\circ,p'_\circ)}(\mathfrak{a}_\circ*\mathfrak{b}_\circ)) &= \sqcup[(\Phi_{\mathcal{W}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ*\mathfrak{b}_\circ),(\Phi_{\mathcal{W}})^{MT}_{\zeta_\circ,p'_\circ}(\mathfrak{a}_\circ*\mathfrak{b}_\circ)]\\ &= \sqcup[\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ*\mathfrak{b}_\circ)+p^{'},\zeta_\circ(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ*\mathfrak{b}_\circ)+p_\circ^{'}]\\ &\geq \sqcup[\zeta(\Phi_{\mathcal{W}})(\mathfrak{b}_\circ)+p^{'},\zeta_\circ(\Phi_{\mathcal{W}})(\mathfrak{b}_\circ)+p_\circ^{'}]\\ &= (\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cup(\zeta_\circ,p'_\circ)}(\mathfrak{b}_\circ).\end{aligned}$$

Thus, $\sqcup[(\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cup(\zeta_\circ,p_\circ)}(\mathfrak{a}_\circ+\mathfrak{b}_\circ),(\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cup(\zeta_\circ,p'_\circ)}(\mathfrak{a}_\circ*\mathfrak{b}_\circ)]$
$\geq \sqcup(\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cup(\zeta_\circ,p'_\circ)}(\mathfrak{a}_\circ),(\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cup(\zeta_\circ,p'_\circ)}(\mathfrak{b}_\circ)$.
And,

$$\begin{aligned}(\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cup(\zeta_\circ,p'_\circ)}(\mathfrak{a}_\circ) &= \sqcup[\zeta(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ)+p^{'},\zeta_\circ(\Phi_{\mathcal{W}})(\mathfrak{a}_\circ)+p_\circ^{'}]\\ &\geq \sqcup[\zeta(\Phi_{\mathcal{W}})(\mathfrak{b}_\circ)+p^{'},\zeta_\circ(\Phi_{\mathcal{W}})(\mathfrak{b}_\circ)+p_\circ^{'}]\\ &= (\Phi_{\mathcal{W}}^{MT})_{(\zeta,p')\cup(\zeta_\circ,p'_\circ)}(\mathfrak{b}_\circ)\end{aligned}$$

In the same manner for the indeterminate and falsity function. Hence, $\mathcal{W}^{MT}_{\zeta,p',q',r'} \cup \mathcal{W}^{MT}_{\zeta_\circ,p'_\circ,q'_\circ,r'_\circ}$ is a neutrosophic subincline(ideal) of $\mathfrak{I}$. □

Definition 3.11. *An mapping f from an incline $\mathfrak{I}$ to $\mathfrak{T}$ and let $\mathcal{W}_{\mathfrak{I}},\mathcal{W}_{\mathfrak{T}}$ be an nmt on $\mathfrak{I}$ & $\mathfrak{T}$ then the preimage of $\mathcal{W}_{\mathfrak{I}}$ of f is, $f^{-1}(\mathcal{W}_{\mathfrak{I}})=\{f^{-1}(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ),f^{-1}(\Psi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,q'}(\mathfrak{a}_\circ)$ $,f^{-1}(\Omega_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ):\mathfrak{a}_\circ\in\mathfrak{I}\}$ such that $f^{-1}(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ)=\Phi_{\mathcal{W}_{\mathfrak{I}}}(\zeta f(\mathfrak{a}_\circ)+p^{'})$, $f^{-1}(\Psi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,q'}(\mathfrak{a}_\circ)=\Psi_{\mathcal{W}_{\mathfrak{I}}}(\zeta f(\mathfrak{a}_\circ)+q^{'})$, $f^{-1}(\Omega_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ)=\Omega_{\mathcal{W}_{\mathfrak{I}}}(\zeta f(\mathfrak{a}_\circ)-r^{'})$.*

Theorem 3.12. *Let $\mathfrak{I}, \mathfrak{T}$ be two inclines and a mapping $f : \mathfrak{I} \to \mathfrak{T}$ be a homomorphism. If the neutrosophic magnified translation $(\mathcal{W}_{\mathfrak{I}})^{MT}_{\zeta,p',q',r'}$ of $\mathfrak{I}$ is an neutrosophic subincline of $\mathfrak{I}$, then $f^{-1}(\mathcal{W}_{\mathfrak{I}})^{MT}_{\zeta,p',q',r'}$ is an neutrosophic subincline of $\mathfrak{I}$.*

Proof. Let the neutrosophic maginified translation $\mathcal{W}_{\mathfrak{I}}$ of $\mathfrak{I}$ be an neutrosophic subincline of $\mathfrak{I}$. Now take $\mathfrak{a}_\circ, \mathfrak{b}_\circ \in \mathfrak{I}$, then

$$\begin{aligned}
\sqcap[f^{-1}(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), f^{-1}(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] &= \sqcap[\zeta f^{-1}(\Phi_{\mathcal{W}_{\mathfrak{I}}})(\mathfrak{a}_\circ + \mathfrak{b}_\circ) + p^{'}, \zeta f^{-1}(\Phi_{\mathcal{W}_{\mathfrak{I}}})(\mathfrak{a}_\circ * \mathfrak{b}_\circ) + p^{'}] \\
&= \sqcap[\zeta(\Phi_{\mathcal{W}_{\mathfrak{I}}})f(\mathfrak{a}_\circ + \mathfrak{b}_\circ) + p^{'}, \zeta(\Phi_{\mathcal{W}_{\mathfrak{I}}})f(\mathfrak{a}_\circ * \mathfrak{b}_\circ) + p^{'}] \\
&\geq \sqcap[\zeta(\Phi_{\mathcal{W}_{\mathfrak{I}}})f(\mathfrak{a}_\circ) + p^{'}, \zeta(\Phi_{\mathcal{W}_{\mathfrak{I}}})f(\mathfrak{b}_\circ) + p^{'}] \\
&= \sqcap[f^{-1}(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ), f^{-1}(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{b}_\circ)].
\end{aligned}$$

Similarly,

$$\begin{aligned}
\sqcap[f^{-1}(\Psi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,q'}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), f^{-1}(\Psi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,q'}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] &\geq \sqcap[f^{-1}(\Psi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,q'}(\mathfrak{a}_\circ), f^{-1}(\Psi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,q'}(\mathfrak{b}_\circ)] \\
\sqcup[f^{-1}(\Omega_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), f^{-1}(\Omega_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] &\leq \sqcup[f^{-1}(\Omega_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ), f^{-1}(\Omega_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,r'}(\mathfrak{b}_\circ)].
\end{aligned}$$

Thus, $f^{-1}(\mathcal{W}_{\mathfrak{I}})^{MT}_{\zeta,p',q',r'}$ is an neutrosophic subincline of $\mathfrak{I}$. □

Theorem 3.13. *A mapping $f : \mathfrak{I} \to \mathfrak{T}$ be a homomorphism of two incline algebra $\mathfrak{I}, \mathfrak{T}$ respectively. If $(\mathcal{W}_{\mathfrak{I}})^{MT}_{\zeta,p',q',r'}$ of $\mathfrak{I}$ is a neutrosophic subincline of $\mathfrak{I}$, then $f(\mathcal{W}_{\mathfrak{I}})^{MT}_{\zeta,p',q',r'}$ is a neutrosophic subincline of $\mathfrak{I}$.*

Proof. Assuming that neutrosophic magnified translation $(\mathcal{W}_{\mathfrak{I}})^{MT}_{\zeta,p',q',r'}$ of $\mathfrak{I}$ is an neutrosophic subincline of $\mathfrak{I}$, then

$$\begin{aligned}
\sqcap[f(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), f(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ * v^{'}_\circ)] &= \sqcap[\zeta f(\Phi_{\mathcal{W}_{\mathfrak{I}}})(\mathfrak{a}_\circ + \mathfrak{b}_\circ) + p^{'}, \zeta f(\Phi_{\mathcal{W}_{\mathfrak{I}}})(\mathfrak{a}_\circ * \mathfrak{b}_\circ) + p^{'}] \\
&= \sqcap[\zeta(\Phi_{\mathcal{W}_{\mathfrak{I}}})f(\mathfrak{a}_\circ + \mathfrak{b}_\circ) + p^{'}, \zeta(\Phi_{\mathcal{W}_{\mathfrak{I}}})f(\mathfrak{a}_\circ * \mathfrak{b}_\circ) + p^{'}] \\
&\geq \sqcap[\zeta(\Phi_{\mathcal{W}_{\mathfrak{I}}})f(\mathfrak{a}_\circ) + p^{'}, \zeta(\Phi_{\mathcal{W}_{\mathfrak{I}}})f(\mathfrak{b}_\circ) + p^{'}] \\
&= \sqcap[f(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ), f(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{b}_\circ)].
\end{aligned}$$

Thus, $\sqcap[f(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ+\mathfrak{b}_\circ), f(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ*\mathfrak{b}_\circ)] \geq \sqcap[f(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ), f(\Phi_{\mathcal{W}_{\mathfrak{I}}})^{MT}_{\zeta,p'}(\mathfrak{b}_\circ)]$. Similarly, for the other terms. Thus, $f(\mathcal{W}_{\mathfrak{I}})^{MT}_{\zeta,p',q',r'}$ is a neutrosophic subincline of $\mathfrak{I}$. □

Definition 3.14. *Let $\mathcal{W}_1, \mathcal{W}_2$ are two nmt subsets of $\mathfrak{I}$ and $\mathfrak{T}$ respectively, then product of $\mathfrak{I}$ and $\mathfrak{T}$ is denoted as $\mathcal{W}_1 \times \mathcal{W}_2$, is written as $\mathcal{W}_1 \times \mathcal{W}_2 = \{(\mathfrak{a}_\circ, \mathfrak{b}_\circ),$ $(\Phi_{\mathcal{W}_1 \times \mathcal{W}_2})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ, \mathfrak{b}_\circ), (\Psi_{\mathcal{W}_1 \times \mathcal{W}_2})^{MT}_{\zeta,q'}(\mathfrak{a}_\circ, \mathfrak{b}_\circ), (\omega_{\mathcal{W}_1 \times \mathcal{W}_2})^{MT}_{\zeta,r'}(\mathfrak{u}_\circ, \mathfrak{b}_\circ) : (\mathfrak{a}_\circ, \mathfrak{b}_\circ) \in \mathfrak{I} \times \mathfrak{T}\}$ where $(\Phi_{\mathcal{W}_1 \times \mathcal{W}_2})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ, \mathfrak{b}_\circ) = \sqcap[\zeta\Phi_{\mathcal{W}_1}(\mathfrak{a}_\circ)+p^{'}, \zeta\Phi_{\mathcal{W}_1}(\mathfrak{b}_\circ)+p^{'}]$, $(\Psi_{\mathcal{W}_1 \times \mathcal{W}_2})^{MT}_{\zeta,q'}(\mathfrak{a}_\circ, \mathfrak{b}_\circ) = \sqcap[\zeta\Psi_{\mathcal{W}_1}(\mathfrak{a}_\circ)+q^{'}, \zeta\Psi_{\mathcal{W}_1}(\mathfrak{b}_\circ)+q^{'}]$, $(\omega_{\mathcal{W}_1 \times \mathcal{W}_2})^{MT}_{\zeta,r'}(\mathfrak{a}_\circ, \mathfrak{b}_\circ) = \sqcup[\zeta\omega_{\mathcal{W}_1}(\mathfrak{a}_\circ)-r^{'}, \zeta\omega_{\mathcal{W}_1}(\mathfrak{b}_\circ)-r^{'}]$.*

Theorem 3.15. *Let $\mathcal{W}_1$ and $\mathcal{W}_2$ be two neutrosophic subincline of $\mathfrak{I}$, then the neutrosophic magnified translation of cartesian product of $(\mathcal{W}_1 \times \mathcal{W}_2)^{MT}_{\zeta,p',q',r'}$ of $\mathcal{W}_1, \mathcal{W}_2$ is an neutrosophic subincline of $\mathfrak{I} \times \mathfrak{I}$.*

Proof. Let $\mathcal{W}_1, \mathcal{W}_2$ be two neutrosophic subincline of $\mathfrak{I}$. By using the theorem 3.4, $(\mathcal{W}_1)^{MT}_{\zeta,p',q',r'}$ and $(\mathcal{W}_2)^{MT}_{\zeta,p',q',r'}$ are neutrosophic subincline of $\mathfrak{I}$. Now,

$$\begin{aligned}
(\Phi_{\mathcal{W}_1\times\mathcal{W}_2})^{MT}_{\zeta,p}(\mathfrak{a}_\circ + \mathfrak{b}_\circ) &= \sqcap[(\Phi_{\mathcal{W}_1})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), (\Phi_{\mathcal{W}_1})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ + \mathfrak{b}_\circ)] \\
&= \sqcap(\zeta\Phi_{\mathcal{W}_1}(\mathfrak{a}_\circ + \mathfrak{b}_\circ) + p^{'}, \zeta\Phi_{\mathcal{W}_2}(\mathfrak{a}_\circ + \mathfrak{b}_\circ) + p^{'}) \\
&\geq \sqcap(\zeta\Phi_{\mathcal{W}_1}(\mathfrak{a}_\circ) + p^{'}, \zeta\Phi_{\mathcal{W}_2}(\mathfrak{a}_\circ) + p^{'}) \\
&= \zeta\Phi_{\mathcal{W}_1\times\mathcal{W}_2}(\mathfrak{a}_\circ) + p^{'} \\
&= (\Phi_{\mathcal{W}_1\times\mathcal{W}_2})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ) \\
(\Phi_{\mathcal{W}_1\times\mathcal{W}_2})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ * \mathfrak{b}_\circ) &= \sqcap[(\Phi_{\mathcal{W}_1})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ * \mathfrak{b}_\circ), (\Phi_{\mathcal{W}_1})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] \\
&= \sqcap(\zeta\Phi_{\mathcal{W}_1}(\mathfrak{a}_\circ * \mathfrak{b}_\circ) + p^{'}, \zeta\Phi_{\mathcal{W}_2}(\mathfrak{a}_\circ * \mathfrak{b}_\circ) + p^{'}) \\
&\geq \sqcap(\zeta\Phi_{\mathcal{W}_1}(\mathfrak{b}_\circ) + p^{'}, \zeta\Phi_{\mathcal{W}_2}(\mathfrak{b}_\circ) + p^{'}) \\
&= \zeta\Phi_{\mathcal{W}_1\times\mathcal{W}_2}(\mathfrak{b}_\circ) + p^{'} \\
&= (\Phi_{\mathcal{W}_1\times\mathcal{W}_2})^{MT}_{\zeta,p'}(\mathfrak{b}_\circ)
\end{aligned}$$

$$\sqcap[(\Phi_{\mathcal{W}_1\times\mathcal{W}_2})^{MT}_{\zeta,p}(\mathfrak{a}_\circ + \mathfrak{b}_\circ), (\Phi_{\mathcal{W}_1\times\mathcal{W}_2})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ * \mathfrak{b}_\circ)] \geq \sqcap[(\Phi_{\mathcal{W}_1\times\mathcal{W}_2})^{MT}_{\zeta,p'}(\mathfrak{a}_\circ), (\Phi_{\mathcal{W}_1\times\mathcal{W}_2})^{MT}_{\zeta,p'}(\mathfrak{b}_\circ)]$$

The proof is similar for the other components. Thus, $(\mathcal{W}_1 \times \mathcal{W}_2)^{MT}_{\zeta,p',q',r'}$ of $\mathcal{W}_1, \mathcal{W}_2$ is a neutrosophic subincline of $\mathfrak{I} \times \mathfrak{I}$. □

4. Conclusion

Neutrosophic set is a third step of fuzzy sets here in this paper consider neutrosophic set to merges with incline algebra which arises as neutrosophic incline sub - algebraic structures. The concept of magnified translation is merged with the neutrosophic structure and is named as neutrosophic magnified translation of incline algebra also discussed with some of its properties such as the preimage, image, product also union, intersection. One can also carried out this to some other kind of algebra or in incline substructures such as r - ideal, regular and so on.

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