J. Indones. Math. Soc.
Vol. 30, No. 03 (2024), pp. 374–384.

GROUP MEAN CORDIAL LABELING OF SOME QUADRILATERAL SNAKE GRAPHS

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Abstract. Let G be a (p, q) graph and let A be a group. Let $f : V(G) \longrightarrow A$ be a map. For each edge uv assign the label $\left\lfloor \frac{o(f(u))+o(f(v))}{2} \right\rfloor$. Here $o(f(u))$ denotes the order of $f(u)$ as an element of the group A. Let $\mathbb{I}$ be the set of all integers labeled by the edges of G. f is called a group mean cordial labeling if the following conditions hold: (1) For $x, y \in A$, $|v_f(x) - v_f(y)| \leq 1$, where $v_f(x)$ is the number of vertices labeled with x. (2) For $i, j \in \mathbb{I}$, $|e_f(i) - e_f(j)| \leq 1$, where $e_f(i)$ denote the number of edges labeled with i. A graph with a group mean cordial labeling is called a group mean cordial graph. In this paper, we take A as the group of fourth roots of unity and prove that, Quadrilateral Snake, Double Quadrilateral Snake, Alternate Quadrilateral Snake and Alternate Double Quadrilateral Snake are group mean cordial graphs.

Key words and Phrases: Cordial labeling, mean labeling, group mean cordial labeling.

1. INTRODUCTION

Graphs considered here are finite, undirected, and simple.Terms not defined here are used in the sense of Harary [4] and Gallian [3]. Somasundaram and Ponraj [6] introduced the concept of mean labeling of graphs.

Definition 1.1. [6] A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to $\{0, 1, 2, \dots, q\}$ such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd then the resulting edge labels are distinct.

Cahit [2] introduced the concept of cordial labeling.

2010 Mathematics Subject Classification: 05C78
Received: 13-05-2022, accepted: 27-10-2024.

Definition 1.2. [2] Let $f : V(G) \rightarrow \{0, 1\}$ be any function. For each edge xy assign the label $|f(x) - f(y)|$. f is called cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1. Also, the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1.

Ponraj et al. [5] introduced mean cordial labeling of graphs.

Definition 1.3. [5] Let f be a function from the vertex set $V(G)$ to $\{0, 1, 2\}$. For each edge uv assign the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called a *mean cordial labeling* if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $i, j \in \{0, 1, 2\}$, where $v_f(x)$ and $e_f(x)$ respectively denote the number of vertices and edges labeled with x $(x = 0, 1, 2)$. A graph with a mean cordial labeling is called a mean cordial graph.

Athisayanathan et al. [1] introduced the concept of group A cordial labeling.

Definition 1.4. [1] Let A be a group. We denote the order of an element $a \in A$ by $o(a)$. Let $f : V(G) \rightarrow A$ be a function. For each edge uv assign the label 1 if $(o(f(u)), o(f(v))) = 1$or 0 otherwise. f is called a group A Cordial labeling if $|v_f(a) - v_f(b)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x)$ and $e_f(n)$ respectively denote the number of vertices labelled with an element x and number of edges labelled with $n(n = 0, 1)$. A graph that admits a group A Cordial labeling is called a group A Cordial graph.

Motivated by these, we define group mean cordial labeling of graphs.

For any real number x, we denoted by $\lfloor x \rfloor$, the greatest integer smaller than or equal to x and by $\lceil x \rceil$, we mean the smallest integer greater than or equal to x. The quadrilateral snake Q_n is obtained from a path P_n by replacing each edge of the path by a quadrilateral. The Double Quadrilateral snake $D(Q_n)$ consists of two quadrilateral snakes that have a common path. The Alternate Quadrilateral snake $A(Q_n)$ is obtained from a path P_n by replacing every alternate edge of the path by a quadrilateral. The Alternate Double quadrilateral $AD(Q_n)$ is obtained from a path P_n by replacing every alternate edge of the path by two quadrilaterals.

2. MAIN RESULTS

Definition 2.1. *Let G be a (p, q) graph and let A be a group. Let f be a map from $V(G)$ to A. For each edge uv assign the label $\left\lfloor \frac{o(f(u))+o(f(v))}{2} \right\rfloor$. Let $\mathbb{I}$ be the set of all integers that are labels of the edges of G. f is called group mean cordial labeling if the following conditions hold:*

(1) For $x, y \in A$, $|v_f(x) - v_f(y)| \leq 1$, where $v_f(x)$ is the number of vertices labeled with x.
(2) For $i, j \in \mathbb{I}$, $|e_f(i) - e_f(j)| \leq 1$, where $e_f(i)$ denote the number of edges labeled with i.

A graph with a group mean cordial labeling is called a group mean cordial graph.

In this paper, we take the group A as the group $\{1, -1, i, -i\}$ which is the group of fourth roots of unity, that is cyclic with generators i and $-i$.

Example 2.2. *The following is a simple example of a group mean cordial graph.*

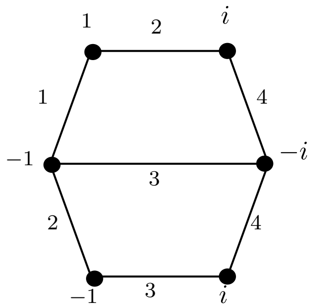

FIGURE 2.1.

Theorem 2.3. *The Quadrilateral Snake Q_n is a group mean cordial graph for every n.*

Proof. Let $P_n : u_1u_2...u_n$ be the path.Let $x_1, x_2, ..., x_{n-1}$; $y_1, y_2, ..., y_{n-1}$ be the newly added vertices. Here $E(Q_n) = \{u_ju_{j+1}, u_jx_j, u_{j+1}y_j, x_jy_j : 1 \leq j \leq n-1\}$. The order and size of the graph are $3n-2$ and $4n-4$.
Define $f : V(Q_n) \longrightarrow \{1, -1, i, -i\}$ as follows:

$$f(u_j) = \begin{cases} 1 & \text{if } j \equiv 1 \quad (mod\, 4) \\ -i & \text{if } j \equiv 0, 2 \quad (mod\, 4) \\ -1 & \text{if } j \equiv 3 \quad (mod\, 4) \end{cases}$$

$$f(x_j) = \begin{cases} -1 & \text{if } j \equiv 1 \quad (mod\, 4) \\ i & \text{if } j \equiv 2 \quad (mod\, 4) \\ 1 & \text{if } j \equiv 3 \quad (mod\, 4) \\ -i & \text{if } j \equiv 0 \quad (mod\, 4) \end{cases}$$

and

$$f(y_j) = \begin{cases} i & \text{if } j \equiv 1 \quad (mod\, 4) \\ 1 & \text{if } j \equiv 2 \quad (mod\, 4) \\ i & \text{if } j \equiv 3 \quad (mod\, 4) \\ -1 & \text{if } j \equiv 0 \quad (mod\, 4) \end{cases}$$

Here $e_f(s) = n-1, \forall\, s \in \{1, 2, 3, 4\}$. Also Table 2.1. proves the vertex condition.

TABLE 2.1.

Nature of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$
$n \equiv 0\,(mod\,4)$	$\frac{3n}{4}$	$\frac{3n}{4}-1$	$\frac{3n}{4}$	$\frac{3n}{4}-1$
$n \equiv 1\,(mod\,4)$	$\frac{3n+1}{4}$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$
$n \equiv 2\,(mod\,4)$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$
$n \equiv 3\,(mod\,4)$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-5}{4}$

Hence the Quadrilateral Snake is a group mean cordial graph. □

Example 2.4. *Group mean cordial labeling of Q_7 is given in Figure 2.2..*

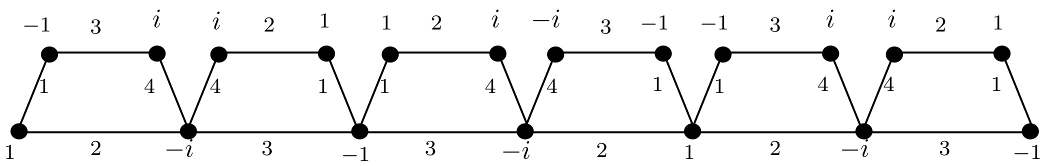

FIGURE 2.2.

Theorem 2.5. *The Double Quadrilateral Snake $D(Q_n)$ is a group mean cordial graph for every n.*

Proof. Let $P_n : u_1u_2...u_n$ be the path.Let $V(D(Q_n)) = V(P_n) \cup \{x_j, y_j, x'_j, y'_j : 1 \leq j \leq n-1\}$. Then $E(D(Q_n)) = E(P_n) \cup \{u_jx_j, u_jx'_j, u_{j+1}y_j, u_{j+1}y'_j, x_jy_j, x'_jy'_j : 1 \leq j \leq n-1\}$. The order and size of the graph are $5n-4$ and $7n-7$.

Case 1: $n \equiv 1\,(mod\,4)$

Define $f : V(D(Q_n)) \longrightarrow \{1, -1, i, -i\}$ as follows:

$$f(u_j) = \begin{cases} 1 & \text{if } j \equiv 0 \quad (mod\,4) \\ -1 & \text{if } j \equiv 1 \quad (mod\,4) \\ i & \text{if } j \equiv 2 \quad (mod\,4) \\ -i & \text{if } j \equiv 3 \quad (mod\,4) \end{cases}$$

$$f(x_j) = f(x'_j) = \begin{cases} -1 & \text{if } j \equiv 0 \quad (mod\,4) \\ i & \text{if } j \equiv 1 \quad (mod\,4) \\ -i & \text{if } j \equiv 2 \quad (mod\,4) \\ 1 & \text{if } j \equiv 3 \quad (mod\,4) \end{cases}$$

and

$$f(y_j) = f(y'_j) = \begin{cases} 1 & \text{if } j \equiv 0 \quad (mod\,4) \\ -1 & \text{if } j \equiv 1 \quad (mod\,4) \\ i & \text{if } j \equiv 2 \quad (mod\,4) \\ -i & \text{if } j \equiv 3 \quad (mod\,4) \end{cases}$$

Case 2: $n \equiv 2\,(mod4)$
Assign the labels to the vertices $u_j(1 \leq j \leq n-1)$ and $x_j, y_j, x'_j, y'_j(1 \leq j \leq n-2)$ as in case 1. Next define $f(u_n) = -i, f(x_{n-1}) = 1, f(x'_{n-1}) = -1$ and $f(y_{n-1}) = f(y'_{n-1}) = i$.
Case 3: $n \equiv 3\,(mod4)$
Assign the labels to the vertices $u_j(1 \leq j \leq n-1)$ and $x_j, y_j, x'_j, y'_j(1 \leq j \leq n-2)$ as in case 2 by replacing n by $n-1$. Next define $f(u_n) = -1, f(x_{n-1}) = i, f(x'_{n-1}) = -i$ and $f(y_{n-1}) = f(y'_{n-1}) = 1$.
Case 4: $n \equiv 0\,(mod4)$
Assign the labels to the vertices $u_j(1 \leq j \leq n-3)$ and $x_j, y_j, x'_j, y'_j(1 \leq j \leq n-4)$ as in case 1. Next define, $f(u_{n-2}) = 1, f(u_{n-1}) = i, f(u_n) = -1; f(x_{n-3}) = f(x'_{n-3}) = -i; f(x_{n-2}) = f(x'_{n-2}) = -1; f(x_{n-1}) = f(x'_{n-1}) = -i$ and $f(y_{n-3}) = i, f(y'_{n-3}) = 1; f(y_{n-2}) = f(y'_{n-2}) = i$. Finally define $f(y_{n-1}) = f(y'_{n-1}) = 1$.
The vertex and edge conditions are established by the following Tables 2.2. & 2.3.

Table 2.2.

Nature of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$
$n \equiv 0\,(mod\,4)$	$\frac{5n-4}{4}$	$\frac{5n-4}{4}$	$\frac{5n-4}{4}$	$\frac{5n-4}{4}$
$n \equiv 1\,(mod\,4)$	$\frac{5n-5}{4}$	$\frac{5n-1}{4}$	$\frac{5n-5}{4}$	$\frac{5n-5}{4}$
$n \equiv 2\,(mod\,4)$	$\frac{5n-6}{4}$	$\frac{5n-2}{4}$	$\frac{5n-2}{4}$	$\frac{5n-6}{4}$
$n \equiv 3\,(mod\,4)$	$\frac{5n-3}{4}$	$\frac{5n-3}{4}$	$\frac{5n-3}{4}$	$\frac{5n-7}{4}$

Table 2.3.

Nature of n	$e_f(1)$	$e_f(2)$	$e_f(3)$	$e_f(4)$
$n \equiv 0\,(mod\,4)$	$\frac{7n-4}{4}$	$\frac{7n-8}{4}$	$\frac{7n-8}{4}$	$\frac{7n-8}{4}$
$n \equiv 1\,(mod\,4)$	$\frac{7n-7}{4}$	$\frac{7n-7}{4}$	$\frac{7n-7}{4}$	$\frac{7n-7}{4}$
$n \equiv 2\,(mod\,4)$	$\frac{7n-10}{4}$	$\frac{7n-6}{4}$	$\frac{7n-6}{4}$	$\frac{7n-6}{4}$
$n \equiv 3\,(mod\,4)$	$\frac{7n-9}{4}$	$\frac{7n-5}{4}$	$\frac{7n-9}{4}$	$\frac{7n-5}{4}$

□

Example 2.6. *Group mean cordial labeling of $D(Q_6)$ is given in Figure 2.3..*

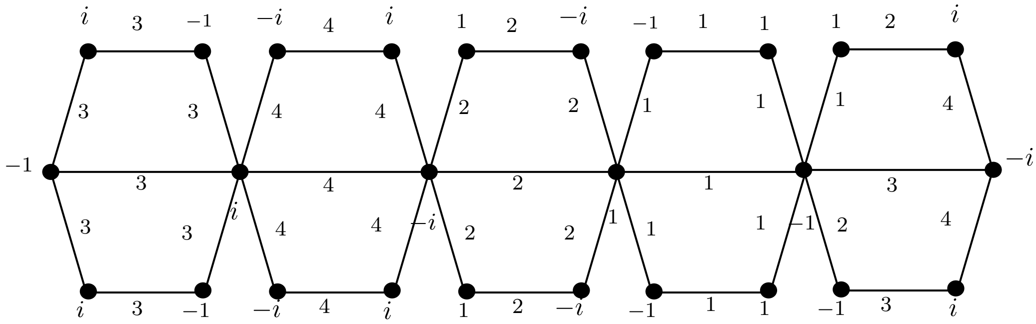

FIGURE 2.3.

Theorem 2.7. *The Alternate Quadrilateral $A(Q_n)$ is a group mean cordial graph for every n.*

Proof. Consider an Alternate quadrilateral graph. It is obtained from a path $P_n : u_1u_2...u_n$ by joining u_j, u_{j+1} (alternatively) to the new vertices x_j, y_j. Then join x_j and y_j. Here every alternate edge of the path is replaced by C_4.

Case 1: The quadrilateral starts from u_1 and the last quadrilateral ends with u_n. Here $|V(A(Q_n))| = 2n$ and $|E(A(Q_n))| = \frac{5n-2}{2}$. Define $f : V(A(Q_n)) \longrightarrow \{1, -1, i, -i\}$ by,

$$f(u_j) = \begin{cases} 1 & \text{if } j \equiv 0,1,4 \quad (mod\, 8) \\ -1 & \text{if } j \equiv 6 \quad (mod\, 8) \\ i & \text{if } j \equiv 2,3,5,7 \quad (mod\, 8) \end{cases}$$

$$f(x_j) = \begin{cases} -1 & \text{if } j \equiv 1 \quad (mod\, 4) \\ -i & \text{if } j \equiv 0,2,3 \quad (mod\, 4) \end{cases}$$

and

$$f(y_j) = \begin{cases} -i & \text{if } j \equiv 1 \quad (mod\, 4) \\ -1 & \text{if } j \equiv 0,2 \quad (mod\, 4) \\ 1 & \text{if } j \equiv 3 \quad (mod\, 4) \end{cases}$$

By this labeling, we get $v_f(1) = v_f(-1) = v_f(i) = v_f(-i) = \frac{n}{2}$. The edge condition is verified by the following Table 2.4.

TABLE 2.4.

Nature of n	$\boldsymbol{e_f(1)}$	$\boldsymbol{e_f(2)}$	$\boldsymbol{e_f(3)}$	$\boldsymbol{e_f(4)}$
$n \equiv 0\, (mod\, 8)$	$\frac{5n}{8} - 1$	$\frac{5n}{8}$	$\frac{5n}{8}$	$\frac{5n}{8}$
$n \equiv 2\, (mod\, 8)$	$\frac{5n-2}{8}$	$\frac{5n-2}{8}$	$\frac{5n-2}{8}$	$\frac{5n-2}{8}$
$n \equiv 4\, (mod\ 8)$	$\frac{5n-4}{8}$	$\frac{5n-4}{8}$	$\frac{5n-4}{8}$	$\frac{5n+4}{8}$
$n \equiv 6\, (mod 8)$	$\frac{5n-6}{8}$	$\frac{5n+2}{8}$	$\frac{5n-6}{8}$	$\frac{5n+2}{8}$

Case 2: The quadrilateral starts from u_2 and the last quadrilateral ends with u_{n-1}.
Here $|V(A(Q_n))| = 2n-2$ and $|E(A(Q_n))| = \frac{5n-8}{2}$. Define $f : V(A(Q_n)) \longrightarrow \{1,-1,i,-i\}$ by,

$$f(u_j) = \begin{cases} 1 & \text{if } j \equiv 1,5 \quad (mod\, 8) \\ -1 & \text{if } j \equiv 2,7 \quad (mod\, 8) \\ i & \text{if } j \equiv 0,3,4,6 \quad (mod\, 8) \end{cases}$$

$$f(x_j) = \begin{cases} 1 & \text{if } j \equiv 1 \quad (mod\, 4) \\ -i & \text{if } j \equiv 0,2,3 \quad (mod\, 4) \end{cases}$$

and

$$f(y_j) = \begin{cases} -i & \text{if } j \equiv 1 \quad (mod\, 4) \\ -1 & \text{if } j \equiv 0,2 \quad (mod\, 4) \\ 1 & \text{if } j \equiv 3 \quad (mod\, 4) \end{cases}$$

The tables 2.5. & 2.6. given below prove that f is a group mean cordial labeling.

Table 2.5.

Nature of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$
$n \equiv 0,4,6\,(mod\,8)$	$\frac{n}{2}$	$\frac{n-2}{2}$	$\frac{n}{2}$	$\frac{n-2}{2}$
$n \equiv 2\,(mod\,8)$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n-2}{2}$	$\frac{n-2}{2}$

Table 2.6.

Nature of n	$e_f(1)$	$e_f(2)$	$e_f(3)$	$e_f(4)$
$n \equiv 0\,(mod\,8)$	$\frac{5n-8}{8}$	$\frac{5n-8}{8}$	$\frac{5n-8}{8}$	$\frac{5n-8}{8}$
$n \equiv 2\,(mod\,8)$	$\frac{5n-2}{8}$	$\frac{5n-10}{8}$	$\frac{5n-10}{8}$	$\frac{5n-10}{8}$
$n \equiv 4\,(mod\,8)$	$\frac{5n-4}{8}$	$\frac{5n-12}{8}$	$\frac{5n-12}{8}$	$\frac{5n-4}{8}$
$n \equiv 6\,(mod\,8)$	$\frac{5n-6}{8}$	$\frac{5n-6}{8}$	$\frac{5n-14}{8}$	$\frac{5n-6}{8}$

Case 3: The quadrilateral starts from u_1 and the last quadrilateral ends with u_{n-1}.
Here $|V(A(Q_n))| = 2n-1$ and $|E(A(Q_n))| = \frac{5n-5}{2}$. Assign the labels to the vertices as in Case 1. Tables 2.7. & 2.8. show that f is a group mean cordial labeling.

Table 2.7.

Nature of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$
$n \equiv 1\,(mod\,8)$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n-1}{2}$	$\frac{n-1}{2}$
$n \equiv 3, 5, 7\,(mod\,8)$	$\frac{n-1}{2}$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$

Table 2.8.

Nature of n	$e_f(1)$	$e_f(2)$	$e_f(3)$	$e_f(4)$
$n \equiv 1\,(mod\,8)$	$\frac{5n-5}{8}$	$\frac{5n-5}{8}$	$\frac{5n-5}{8}$	$\frac{5n-5}{8}$
$n \equiv 3\,(mod\,8)$	$\frac{5n-7}{8}$	$\frac{5n-7}{8}$	$\frac{5n-7}{8}$	$\frac{5n+1}{8}$
$n \equiv 5\,(mod\,8)$	$\frac{5n-9}{8}$	$\frac{5n-1}{8}$	$\frac{5n-9}{8}$	$\frac{5n-1}{8}$
$n \equiv 7\,(mod\,8)$	$\frac{5n-11}{8}$	$\frac{5n-3}{8}$	$\frac{5n-3}{8}$	$\frac{5n-3}{8}$

Case 4: The quadrilateral starts from u_2 and the last quadrilateral ends with u_n. Here $|V(A(Q_n))| = 2n - 1$ and $|E(A(Q_n))| = \frac{5n-5}{2}$. Assign the labels to the vertices as in Case 2. Here $v_f(1) = \frac{n+1}{2}$ and $v_f(-1) = v_f(i) = v_f(-i) = \frac{n-1}{2}$. Table 2.9. proves that f is a group mean cordial labeling.

Table 2.9.

Nature of n	$e_f(1)$	$e_f(2)$	$e_f(3)$	$e_f(4)$
$n \equiv 1\,(mod\,8)$	$\frac{5n-5}{8}$	$\frac{5n-5}{8}$	$\frac{5n-5}{8}$	$\frac{5n-5}{8}$
$n \equiv 3\,(mod\,8)$	$\frac{5n+1}{8}$	$\frac{5n-7}{8}$	$\frac{5n-7}{8}$	$\frac{5n-7}{8}$
$n \equiv 5\,(mod\,8)$	$\frac{5n-1}{8}$	$\frac{5n-9}{8}$	$\frac{5n-9}{8}$	$\frac{5n-1}{8}$
$n \equiv 7\,(mod\,8)$	$\frac{5n-3}{8}$	$\frac{5n-3}{8}$	$\frac{5n-11}{8}$	$\frac{5n-3}{8}$

□

Example 2.8. *Group mean cordial labeling of $A(Q_8)$ is given in Figures 2.4.*

Theorem 2.9. *The Alternate Double Quadrilateral $AD(Q_n)$ is a group mean cordial graph for every n.*

Proof. Let $P_n : u_1u_2...u_n$ be the common path.
Case 1: n is even.
Case 1.1: The alternate double quadrilateral starts from u_2 and the last double quadrilateral ends with u_{n-1}.
Let $V(AD(Q_n)) = V(P_n) \cup \{v_j, w_j, v'_j, w'_j : 1 \leq j \leq \frac{n-2}{2}\}$. Then $E(AD(Q_n)) =$

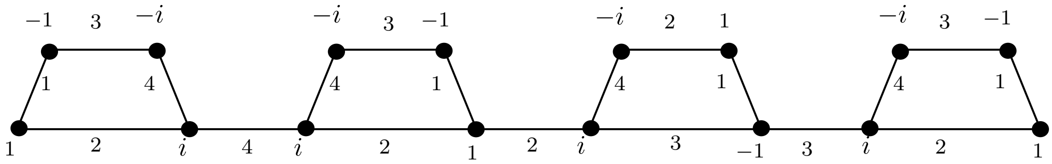

Figure 2.4.

$E(P_n) \cup \{u_{2j}v_j, u_{2j}v'_j, u_{2j+1}w_j, u_{2j+1}w'_j, v_jw_j, v'_jw'_j : 1 \leq j \leq \frac{n-2}{2}\}$. The order and size of the graph are $3n-4$ and $4n-7$.

Define $f : V(AD(Q_n)) \longrightarrow \{1, -1, i, -i\}$ by :

$$f(u_j) = \begin{cases} 1 & \text{if } j \equiv 5,7 \pmod 8 \\ -1 & \text{if } j \equiv 0,3,6 \pmod 8 \\ i & \text{if } j \equiv 1 \pmod 8 \\ -i & \text{if } j \equiv 2,4 \pmod 8 \end{cases}$$

$$f(v_j) = \begin{cases} i & \text{if } j \equiv 1,2,3 \pmod 4 \\ 1 & \text{if } j \equiv 0 \pmod 4 \end{cases}$$

$$f(w_j) = \begin{cases} -1 & \text{if } j \equiv 1,2 \pmod 4 \\ i & \text{if } j \equiv 0,3 \pmod 4 \end{cases}$$

and

$$f(v'_j) = \begin{cases} 1 & \text{if } j \equiv 1 \pmod 4 \\ -1 & \text{if } j \equiv 0 \pmod 4 \\ -i & \text{if } j \equiv 2,3 \pmod 4 \end{cases}$$

$$f(w'_j) = \begin{cases} 1 & \text{if } j \equiv 1,2 \pmod 4 \\ -i & \text{if } j \equiv 0,3 \pmod 4 \end{cases}$$

Tables 2.10. & 2.11. prove that f is a group mean cordial labeling.

Table 2.10.

Nature of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$
$n \equiv 0,4 \pmod 8$	$\frac{3n-4}{4}$	$\frac{3n-4}{4}$	$\frac{3n-4}{4}$	$\frac{3n-4}{4}$
$n \equiv 2 \pmod 8$	$\frac{3n-6}{4}$	$\frac{3n-6}{4}$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$
$n \equiv 6 \pmod 8$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$	$\frac{3n-6}{4}$	$\frac{3n-6}{4}$

TABLE 2.11.

Nature of n	$e_f(1)$	$e_f(2)$	$e_f(3)$	$e_f(4)$
$n \equiv 0, 6 \pmod 8$	$n-1$	$n-2$	$n-2$	$n-2$
$n \equiv 2 \pmod 8$	$n-2$	$n-2$	$n-2$	$n-1$
$n \equiv 4 \pmod 8$	$n-2$	$n-2$	$n-1$	$n-2$

Case 1.2: The alternate double quadrilateral starts from u_1 and the last double quadrilateral ends with u_n.
Let $V(AD(Q_n)) = V(P_n) \cup \{v_j, w_j, v'_j, w'_j : 1 \leq j \leq \frac{n}{2}\}$. Then $E(AD(Q_n)) = E(P_n) \cup \{u_{2j-1}v_j, u_{2j-1}v'_j, u_{2j}w_j, u_{2j}w'_j, v_jw_j, v'_jw'_j : 1 \leq j \leq \frac{n}{2}\}$. The order and size of the graph are $3n$ and $4n-1$.
Define $f : V(AD(Q_n)) \longrightarrow \{1, -1, i, -i\}$ by :

$$f(u_j) = \begin{cases} i & \text{if } j \equiv 1 \pmod 4 \\ -1 & \text{if } j \equiv 2 \pmod 4 \\ -i & \text{if } j \equiv 3 \pmod 4 \\ 1 & \text{if } j \equiv 0 \pmod 4 \end{cases}$$

$f(v_j) = i$ and $f(v'_j) = -i$, for all j.

$$f(w_j) = f(w'_j) = \begin{cases} 1 & \text{if } j \equiv 1 \pmod 2 \\ -1 & \text{if } j \equiv 0 \pmod 2 \end{cases}$$

The vertex and edge conditions are satisfied by the following Tables 2.12. & 2.13.

TABLE 2.12.

Nature of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$
$n \equiv 0 \pmod 4$	$\frac{3n}{4}$	$\frac{3n}{4}$	$\frac{3n}{4}$	$\frac{3n}{4}$
$n \equiv 2 \pmod 4$	$\frac{3n+2}{4}$	$\frac{3n-2}{4}$	$\frac{3n+2}{4}$	$\frac{3n-2}{4}$

TABLE 2.13.

Nature of n	$e_f(1)$	$e_f(2)$	$e_f(3)$	$e_f(4)$
$n \equiv 0 \pmod 4$	n	$n-1$	n	n
$n \equiv 2 \pmod 4$	n	n	$n-1$	n

Case 2: n is odd.
Here the order and size of the graph are $3n-2$ and $4n-4$.
Case 2.1: The alternate double quadrilateral starts from u_2 and the last double quadrilateral ends with u_n.
Label the vertices as in subcase 1.1. Here $e_f(s) = n-1$, for all $s \in \{1, 2, 3, 4\}$. Table 2.14. proves the vertex condition.

Table 2.14.

Nature of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$
$n \equiv 1 \pmod 8$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$	$\frac{3n+1}{4}$	$\frac{3n-3}{4}$
$n \equiv 3 \pmod 8$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-5}{4}$
$n \equiv 5 \pmod 8$	$\frac{3n+1}{4}$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$
$n \equiv 7 \pmod 8$	$\frac{3n-1}{4}$	$\frac{3n-5}{4}$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$

Case 2.2: The alternate quadrilateral starts from u_1 and the last quadrilateral ends with u_{n-1}.
Label the vertices as in subcase 1.2. Here also, $e_f(s) = n - 1$, for all $s \in \{1, 2, 3, 4\}$.Table 2.15. proves the vertex condition.

Table 2.15.

Nature of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$
$n \equiv 1 \pmod 4$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$	$\frac{3n+1}{4}$	$\frac{3n-3}{4}$
$n \equiv 3 \pmod 4$	$\frac{3n-1}{4}$	$\frac{3n-5}{4}$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$

Hence $AD(Q_n)$ is a group mean cordial graph for all n. □

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