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EFFECT OF COVID-19 PATIENTS FLOW RATE FROM ER TO LPD ON COVID-19 PATIENTS HEALING RATES AT RSUP DR. SARDJITO'S ER

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Abstract COVID-19 or Coronavirus Disease-19 which first appeared in December 2019 has spread around the world, one of which is Indonesia. The spread of the COVID-19 has affected all areas of human life and the most affected area is health. It can be known from the number of COVID-19 patients in numerous national referral hospitals, one of which is RSUP Dr. Sardjito. This large number of COVID-19 patients often causes a high number of bed occupancy rates and has an impact on special rooms, the room for COVID-19 patients who come to the hospital for the first time or the Emergency Room (ER) and the room for COVID-19 patients from the ER who is declared seriously ill or inpatient (Long-stay Patient Department, LPD). Furthermore, imbalance between the number of ER, LPD, and the number of COVID-19 patients yield the flow rate of COVID-19 patients from the ER to the LPD is decreasing. To find out the impact of flow rate, this research is conducted by using Richard's Curve and Kaplan-Meier Estimator. The results of this research show effect of flow rate which can be observed from the probability of COVID-19 patients who recovered or died in each special room. The probability of a COVID-19 patient in the ER recovered is greater than the probability of a patient died. Meanwhile, the probability of a COVID-19 patient in the LDP recovered is smaller than the probability of a patient died.

Key words and Phrases: COVID-19, flow rate, Richard's curve, Kaplan-Meier estimator, bed occupancy rate.

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1. INTRODUCTION

Pandemic COVID-19 impacts to many hospitals worldwide having crisis on his management or infrastructure facilities in giving the service. The rise of the COVID-19 case impacted the specialized room space as Emergency Room (ER) and Long-stay Patient Department (LPD). The ER is a room for patient COVID-19 that comes to the hospital for the first time, and then the patient will be classified by the medical officer who became the acute patient or not, whereas LPD is the room for patient COVID-19 in serious condition from the emergency room. The specialization room as ER and LPD has a maximum capacity, so if the case of COVID-19 rising very fast, it causes an accumulation of patients or flow rate patient COVID-19 from the ER to the LPD will increase.

The purpose of the research is to answer several problems, among to arrange and analyze the mathematical model of the flow rate patient COVID-19 from the ER to the LPD RSUP Dr. Sardjito, calculate the death or recovery probability of the patient COVID-19 in the ER and LPD. In addition, the research aims to estimate the transfer probability of patient COVID-19 to the LPD. The research seeks to determine the probability distributions of the total patient waiting to transfer to the ER or LPD and investigate side-effects of flow rate patient COVID-19 in the ER to LPD toward the recovery percentage patient COVID-19 in RSUP Dr. Sardjito.

Meanwhile, the benefit of this research is to provide an information related to the impact of patient COVID-19's flow rate from the ER to the LPD. This information can be utilized by hospital administrators to improve their handling of Covid-19, reducing the number of pandemic-related deaths. Also, this research is an alarm to the government to make a better management policy of the hospital that there is no more death causes the lateness service by the hospital. Furthermore, this research can be applied in other health facilities.

Focus of this research is to formulate a mathematical model that potentially to be implemented in other hospitals. It can assist the hospital in obtaining information about the impact of patient COVID-19's flow rate from the ER to the LPD on the patient's recovery in the ER. According to Baas, et.al. [1], the model formulation is relatively valid. Furthermore, the topics covered in this research are relevant to the real-world problem, and more research is required to combat the global Covid-19 pandemic.

2. Methods

2.1. **Subject Research.** The research subject is patient referrals COVID- 19 in RSUP Dr. Sardjito with any age category that includes the children to adult categories listed on 1 March to 1 April 2021 period.

2.2. **Method Data Collection.** The data is undertaken from Medical Record Installation (ICM) RSUP Dr. Sardjito on 1 March to 1 April 2021. Data of patient COVID-19 taken among are the number of patients to the ER, the number of patients from the ER include the patients recovery, the patient died and the transferred patient to the LPD. Besides, data of the number of patients go into LPD and the number of patients exit to the LPD include the patients recovery and the patient died.

2.3. **Analysis Method.** In this research, the probabilistic model is carried out using related probability theories and an analytic model of population growth and mortality. The main purpose is to calculate the number of COVID-19 patients in the ER and LPD Dr. Sardjito.

2.3.1. *Probability of the patient being transferred to ER or LPD.* The probabilistic model is determined by using the Kaplan-Meier estimator. First, sorting the data that has been taken to get data on patients living in the ER and LPD at each period. Then, from the sorted data, the probability of the patient being transferred cumulatively can be determined by using the Kaplan-Meier estimator. Kaplan-Meier estimator is given as follows

$$F(l) = 1 - \prod_{u=1}^{l} \left(1 - \frac{e(u)}{n(u)}\right), \tag{1}$$

where $F(l), l, e(u)$, and $n(u)$ represent the probability of the patient staying in the ER or LPD, the lenght of time, the number of patients at time $t-1$, and the number of patients at time t, respectively.

In this research, the complement of these probabilities is the cumulative probability that the patient will be transferred to the ER or LPD. This probability model must empirically divide the number of patients transferred at time $t-1$ by the current number of patients at time t. Then, the probability that the patient will be transferred is

$$T(t) = \frac{1 - F_{emp}(t)}{1 - F(t)}, \tag{2}$$

where $F_{emp}(t)$ represents the cumulative probability of patients being transferred at time t.

2.3.2. *Probability of the patient recovery.* In determination of the probability distribution of recovered patients is influenced by the queueing factor in the ER or LPD. First, fitting the data using Weibull's model to get an estimation of the cumulative number of cured patients at any time t. Weibull's model is given as follows

$$s(t) = a(1 - e^{-(bt)^c}), \tag{3}$$

where $s(t)$ represents the number of recovered patients, whereas a, b, and c denote the upper asymptote parameter, the scale parameter, and the shape parameter of the distribution, respectively.

Then, by using Equation (3), the probability of the patient recovery cumulatively over time t is

$$P_{s_i}(t \leq s) = 1 - e^{-(bt)^c}. \tag{4}$$

Furthermore, by differentiating Equation (4) with respect to time t, it is obtained the probability distribution of patients recovery as follows

$$P_s(t = s) = bc(bt)^{c-1}e^{-(bt)^c}. \tag{5}$$

2.3.3. *Probability of patient dying.* In determination of the probability distribution of patients dying is equivalently as determining the probability distribution of patients recovery. The estimated number of patients who died each time cumulatively is given as

$$m(t) = a_1(1 - e^{-(b_1 t)^{c_1}}), \tag{6}$$

where $m(t)$ represents the number of died patients, whereas a_1, b_1, and c_1 denote the upper asymptote, the scale parameter, and the shape parameter of the distribution, respectively.

Then, by using Equation (6), the cumulative probability of the patient dying at any time is

$$P_{m_i}(t \leq s) = 1 - e^{-(b_1 t)^{c_1}}. \tag{7}$$

Furthermore, by differentiating Equation (7) with respect to time t, the probability distribution of patients dying is

$$P_s(t = s) = b_1 c_1 (b_1 t)^{c_1 - 1} e^{-(b_1 t)^{c_1}}. \tag{8}$$

2.3.4. *Cumulative patient arrivals.* Richard's Curve is used to determine the overall number of patients in the ER and LPD in each period. First, the data is sorted to get the number of patients who enter each room in each period. Then fitting the data using Richard's Curve, which is defined as

$$N(t) = \int_{\infty}^{t} \lambda(s)ds = \frac{R - L}{(q + \delta exp(-kt))^{\frac{1}{\delta}}} + L, \tag{9}$$

where R, L, k, and δ represent the upper asymptote, the lower asymptote, the growth rate, and the affects near maximum asymptote, respectively.

Clearly that the patient admission rate $\lambda(s)$ can be determined by differentiating $N(t)$ with respect to time t. The following equation is obtained

$$\lambda(t) = \frac{dN(t)}{dt}. \tag{10}$$

2.3.5. *Probability of the number of patients queuing to be transferred to ER or LPD.* Poisson distribution is applied to determine the probability of the number of patients waiting in line to be transferred with parameter ρ_R in ER and ρ_I in LPD. Based on the data taken in the period, that is 1 March to 1 April 2021, it is obtained that the interarrival times of COVID-19 patients at Dr. Sardjito follows an exponential distribution, that is the arrival process is a Poisson process. The Poisson distribution is defined as

$$P(n_i(t)) = \frac{e^{\rho_i(t)}\rho(t)^{n_i(t)}}{n_i(t)!}, \quad \text{for} \quad i = I, R. \tag{11}$$

3. MAIN RESULTS

In this research, the probabilistic model is formulated by using related probability theories and an analytical model of the population growth and mortality. Poisson distribution is used to determine the probability of a patient leaving the ER and LPD. Then, fitting the data to get the model parameters, according to the data.

3.1. **Mathematical Model.**

In this research, we consider some assumptions as follow:

a. There are two service rooms for COVID-19 patients.
b. COVID-19 patients can enter the hospital only through the ER.
c. COVID-19 patients in the ER can be transferred to LPD if their condition is severe.
d. LPD COVID-19 patients can be transferred to the ER if their condition improves.
e. There is a queue of COVID-19 patients to be transferred.
f. COVID-19 patients can leave each room in a state of death or recovery.

From Equation (11), the random variable is the number of patients waiting in line and the Poisson distribution parameter $\rho(t)$ depends on time t. The Poisson distribution parameters in each of the probability models of the ER and LPD are obtained from the numerical solution of the system of differential equations

$$\begin{cases} \dfrac{d\rho_I}{dt} = & T_R(t)\rho_R(t) - T_I(t)\rho_R \\ \dfrac{d\rho_R}{dt} = & \lambda(t) + T_I(t)\rho_I(t) - T_R(t)\rho_R(t). \end{cases} \tag{12}$$

Further, the system of differential Equation (12) is solved numerically by Euler Method to get Poisson distribution parameters ρ_I and ρ_R. The Poisson distribution of the number of patients queueing in the ER and LPD is obtained as follow

$$\begin{cases} P_I(n_I(t)) & = \dfrac{e^{\rho_I(t)}\rho_I(t)^{n_I(t)}}{n_I(t)!} \\ P_R(n_R(t)) & = \dfrac{e^{\rho_R(t)}\rho_R(t)^{n_R(t)}}{n_R(t)!} \\ n_I & = (1 - T_I)(N_I(t+1) - N_I(t)) \\ n_R & = (1 - T_R)(N_R(t+1) - N_R(t)), \end{cases} \tag{13}$$

where n_i, T_i, Ps_i, P_{m_i}, and N_i, for $i =$ I, R, represent the number of patients waiting in i, the probability of patients transfer in i, patients recovery in i, patients dying in i, and the number of patients in i, respectively. Transition diagram of Equation (12) is given in 1.

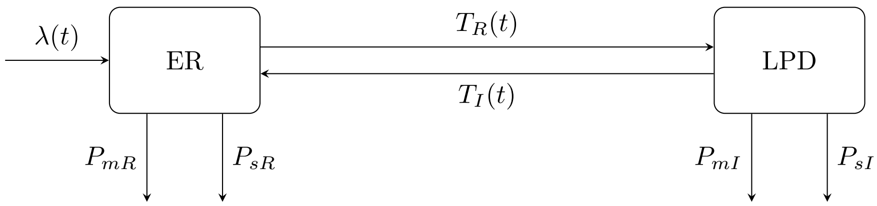
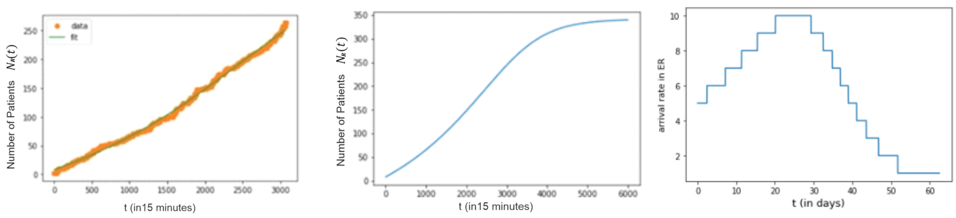


FIGURE 1. Transition diagram of COVID-19 patient

3.2. **Cumulative Number of Patients in the ER and LPD.** The value of the parameters of ER can be carried out by using Richard's Curve (see Figure 2a), where $R = 340.726$, $L = -101.61$, $k = 0.001$, and $\delta = 3.812$. There are 350 patients who have been admitted to the ER if there are no patients who are discharged and transferred during the time interval $t = 0$ to $t = 4000$ (in 15 minutes), see Figure 2b. Figure 2c depicts the maximum arrival rate that can reach 10 patient per day and the average patient arrival rate is 9 people per day. The time t is extended to investigate the behaviour of cumulative number of patients in ER and LPD, namely N_I and N_R, respectively.

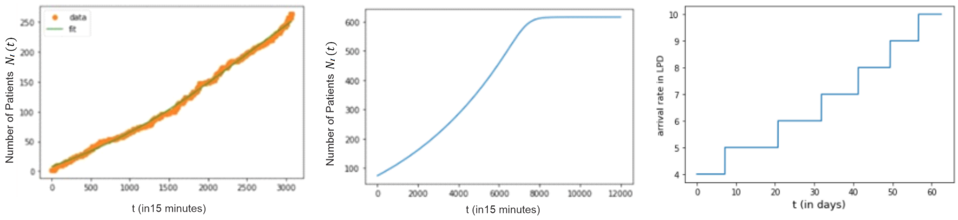


(A) (B) (C)

FIGURE 2. (A) Fitting data of patients in the ER, (B) Result of fitting data of patients in the ER, and (C) Arrival rate in the ER

Equivalently, the value of the parameters of LPD can be carried out by using Richard's Curve (see Figure 3a), where $R = 615.009$, $L = -141.8$, $k = 0.003$, and $\delta = 19.825$. There are 600 patients who have been admitted to the LPD if during the time interval $t = 0$ to $t = 8000$, see Figure 3b. Figure 3c depicts the maximum arrival rate that can reach 10 patient per day and the average patient arrival rate is 9 people per day. The time t is extended to investigate the behaviour of cumulative number of patients in ER and LPD, namely N_I and N_R, respectively.

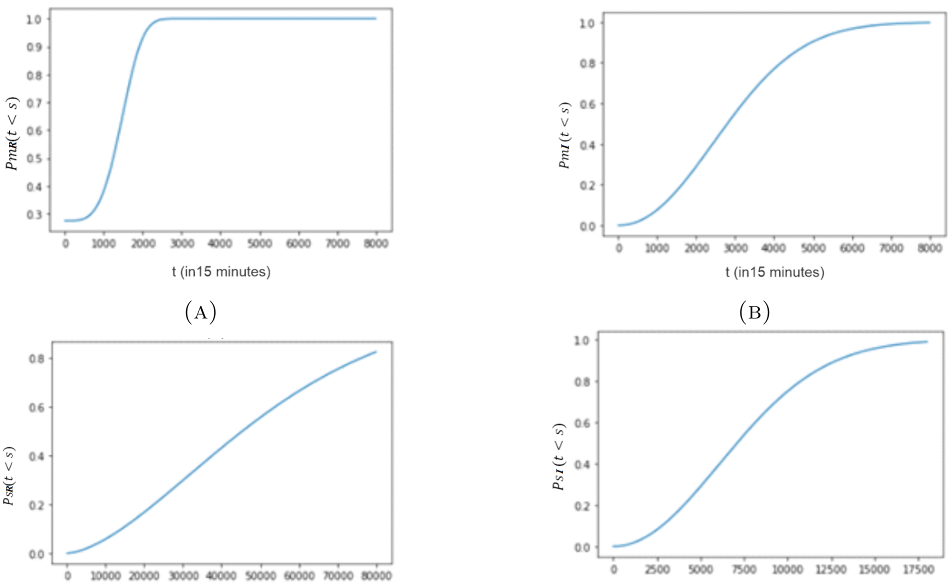
Further, it can be concluded that the model is stable towards the number of patients of 350 in the ER and 600 in the LPD. In this case, patients in the ER and LPD have a probability of recovery, dying, and being transferred to the ER or LPD.



(A) (B) (C)

FIGURE 3. (A) Fitting data of patients in the LDP, (B) Result of fitting data of patients in the LPD, and (C) Arrival rate in the LPD

3.3. **Probability of Patients Dying or Recovering in ER and LPD.** The cumulative probability of patient recovery and dying in the ER and LPD generated from fitting the data by using Weibull's model. In this model there is a maximum number of recoveries and deaths based on data obtained in the period 1 March to 1 April 2021 at Dr. Sardjito Centre General Hospital Yogyakarta.



(A) (B) (C) (D)

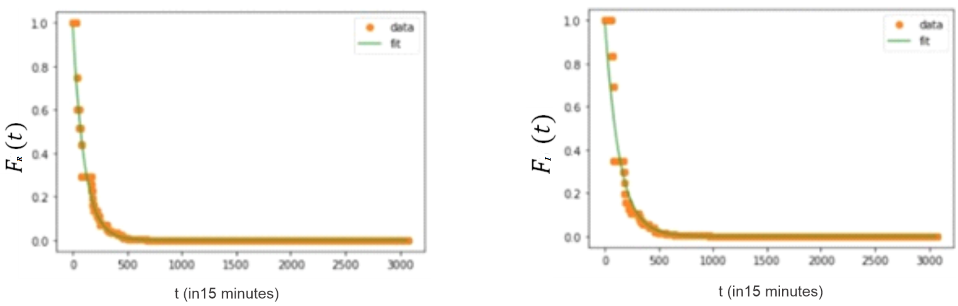
FIGURE 4. Probability of the patient dying/recovering in ER and LPD

For the probability of the patients who died in the ER and LPD, it can be seen in Figure 4a and 4b, while for recovered patients in the ER and LPD, it can be seen in Figure 4c and 4d. Based on Figure 4, it can be seen from each room that the probability of the number of patients dying will reach the maximum number in the data faster than the probability of the number of patient recovery. In the next subsection, we will discuss the probability of a patient being transferred to the ER or LPD.

3.4. **Probability of a Patient Being Transferred to ER or LPD.** In this research, the Kaplan Meier estimator is used to determine the probability of a patient waiting analytically. Since the inter-arrival time is exponentially distributed, the complement of the cumulative distribution function satisfies the following equation

$$P(T > t + s \mid T > s) = \frac{P(T > t + s)}{P(T > s)} = \frac{e^{-\alpha(s+t)}}{e^{-\alpha s}} = P(T > t). \tag{14}$$

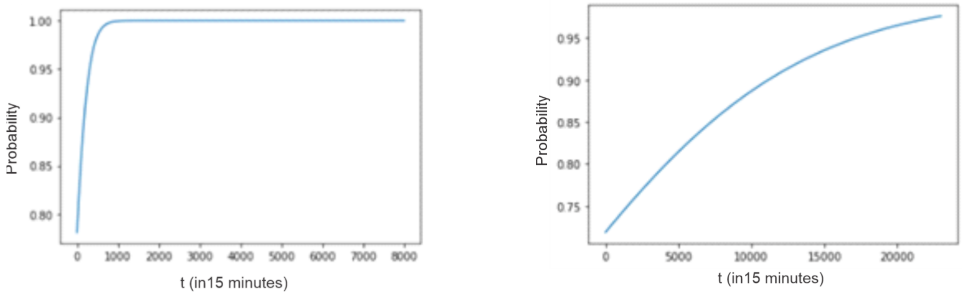
In this case, the probability of a patient waiting is complementary to the probability of a patient being transferred to the ER or LPD. To get the parameter values, data fitting is carried out on the data that has been processed with the Kaplan Meier estimator. The results of fitting the data can be seen in Figure 5a and Figure 5b.



(A) (B)

Figure 5. Fitting data with Kaplan-Meier Estimator

Furthermore, to get the probability of a patient being transferred, we divide the complement of the probability of a patient waiting analytically with the complement of the probability of a patient waiting empirically. The probability of a patient being transferred can be seen in Figures 6a and 6b.

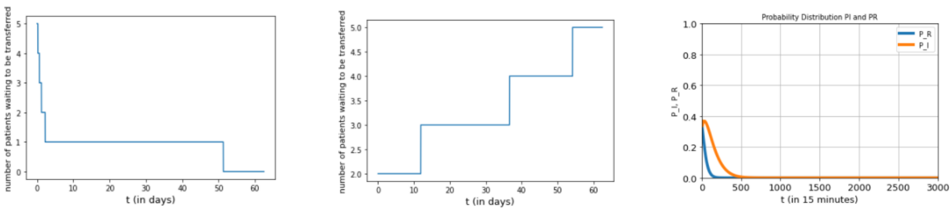


(A) (B)

Figure 6. (A) Probability of a patient being transferred from ER and (B) Probability of a patient being transferred from LPD

Note that the probability of a patient being transferred from ER to LPD is greater than the probability of a patient being transferred from LPD to ER. This can be interpreted by the flow of patients from ER to LPD greater than the flow of patients from LPD to ER which resulted in the number of patients living in LPD is much greater than the number of patients living in ER. Next subsection discusses the probability distribution of the number of patients waiting to be transferred to the ER or LPD.

3.5. **Probability Distributions of The Number of Patients Waiting to Be Transferred to the ER or LPD.** To determine the probability distribution of the number of patients waiting to be transferred to the ER or LPD used the Poisson distribution. Poisson distribution parameter in ER and LPD are denoted by $\rho_R(t)$ and $\rho_I(r)$, respectively. The probability distribution of the number of patients waiting to be transferred to the ER or LPD is shown in Figure 7.



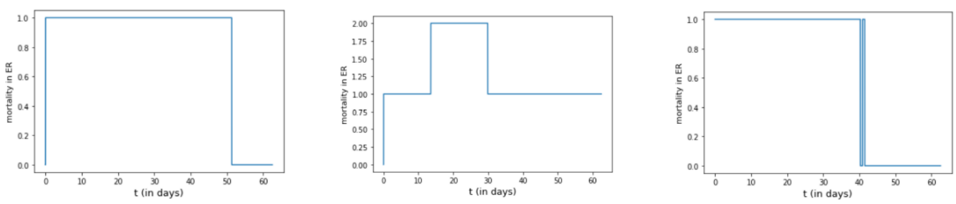
(A) (B) (C)

Figure 7. The probability of the number of patients will be transferred to the ER or LPD

Note that in Figure 7a and Figure 7b, the number of patients admitted to ER continues to decrease because the transfer rate from ER to LPD is quite large, while the number of patients admitted to LPD continues to increase because the rate of transfer from LPD to ER is not too large. Then, the distribution of the number of patients waiting to be transferred from ER and LPD shown in Figure 7c. It is seen that the highest probability for 5 patients waiting in ER is 0.4. So, the waiting time of 5 patients is less than 10 hours to be transferred to LPD. Meanwhile, the highest probability of 2 patients waiting in LPD is 0.4. It is later obtained that the average waiting time for COVID-19 patients in ER is 5.046 hours.

In the next subsection will be discussed about the simulation model of the effect of flow rate on death and recovery of patients.

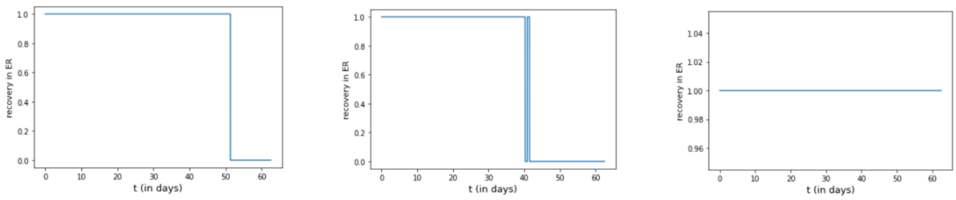
3.6. **Simulation models of the effect of transfer function (flow rate) on death and recovery of patients.** In this simulation will be investigated the effect of patient transfer function on death and recovery of patients. The simulation is conducted using a transfer function based on the data obtained, a transfer function that is enlarged with a value on α to 3.9 and a transfer function that is reduced by a value in α to 0.001. In this case, if the transfer function is enlarged, the possibility of the patient being transferred is greater so that the number of patients transferred will increase, while if the transfer function is reduced then the possibility of patients being transferred is smaller so that the number of patients transferred will decrease. The number of mortality each day in ER with transfer function based on the data obtained can be observed in Figure 8a. The number of patients dying with reduced and enlarged transfer functions shown in Figure 8b and Figure 8c.



(A) (B) (C)

Figure 8. Graph of mortality each day in ER simulation results

Note that if the transfer function is reduced, then the number of mortality increase. So, the number of patient deaths increases considerably. Meanwhile, if the transfer function is enlarged, then the number of mortality tends to decrease. The recovery of patients in ER depicted in Figures 9a, 9b and 9c. Further, Figure 9a, 9b, and 9c depict the number of recovery with the transfer function based on the data obtained, the number of recovery with a reduced transfer function, and the number of recovery with an enlarged transfer function, respectively.



(A) (B) (C)

FIGURE 9. Graph of recovery each day in ER simulation results

It can be conclude that if the transfer function is reduced, then the number of recovery will decrease slightly when compared to the recovery of patients with the transfer function of actual data. Meanwhile, if the transfer function is enlarged, then the number of recovery will increase slightly when compared to the number of patients with the transfer function of actual data.

4. CONCLUDING REMARKS

This research is conducted by processing COVID-19 patient data at Central General Hospital Dr. Sardjito in the period 1 March to 1 April 2021. The average waiting time of COVID-19 patients in ER is 302.76 minutes or equivalent to 5.046 hours, with the average arrival rate is 9 people per day, whereas the maximum number of patients treated in ER is 5 peoples in one day and the number decreases because the rate of patients recovered or transferred to LPD is quite high.

We have shown that there is an effect of the transfer function (flow rate) on the death of patients in ER. Further, the flow rate has an effect on the cure rate of COVID-19 patients in ER. Based on the flow rate $T_R(t)$, the number of patients who died is 52 patients in two months with maximum mortality rate is 1 patient per day and the number of patients who recovering is 52 patients in two months with maximum number recovery rate is 1 patient per day. Furthermore, two cases have been considered based on the value of the flow rate obtained. If the flow rate of $T_R(t)$ is reduced, then the prediction of the number of patient deaths is 75 patients in two months with the maximum mortality rates is 2 people per day and the number of patient cures is 41 patients in two months with the maximum recovery rate is 1 patient per day. If the flow rate of $T_R(t)$ is enlarged, then the number of patient deaths is 42 patients with the maximum mortality rate is 1 patient per day and the number of patient cures is 60 patients with the maximum recovery rate is 1 patient per day.

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